

CHAOTIC BEHAVIOR OF BOUNCING SYSTEMS

BY

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To My Mother, Won-Ae Ro, who passed away in 1981

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TABLE OF CONTENTS

	<u>Page</u>
ACKNOWLEDGEMENT	iii
ABSTRACT	vi
CHAPTERS	
1. INTRODUCTION	1
1.1 Problem Statement and Previous Work	1
1.2 Chaotic Systems	7
1.3 The Goal and Organization of the Work	12
2. SIMPLIFIED BOUNCING SYSTEMS	14
2.1 Background	14
2.2 One Degree of Freedom Spring-Mass System	15
2.3 Phase Planes of One Degree of Freedom Bouncing Systems	16
2.4 Two Degree of Freedom Spring-Mass Systems	20
2.5 Phase Planes of Two Degree of Freedom Bouncing Systems	22
3. CHAOS IN TWO DEGREE OF FREEDOM BOUNCING SYSTEMS	27
3.1 Background	27
3.2 Sum of Standard Deviations and Area in Phase Plane Plot for the Specified Region	30
3.3 Spectrum Analyses for the Chaotic Bouncing Systems with $h = 0.2m$	35
3.3.1 Case of 570N/m as Foot Stiffness (K_2)	35
3.3.2 Case of 3670N/m as Foot Stiffness (K_2)	42
3.4 Spectrum Analyses for the Chaotic Bouncing Systems with $h = 0.5m$	47
3.4.1 Case of 445N/m as Foot Stiffness (K_2)	48
3.4.2 Case of 1860N/m as Foot Stiffness (K_2)	52

3.5 Spectrum Analyses for the Chaotic Bouncing Systems with $h = 1.0m$	60
3.5.1 Case of 2010N/m as Foot Stiffness (K_2)	60
3.5.2 Case of 2745N/m as Foot Stiffness (K_2)	66
3.6 Summary	72
4. ELIMINATION OF CHAOS IN TWO DEGREE OF FREEDOM SYSTEMS	81
4.1 Background	81
4.2 Spring Selections	81
4.3 Use of Damping Elements	84
5. CONCLUSIONS AND RECOMMENDATIONS	90
APPENDICES	
A. EFFECTS OF TIME STEP SIZE	92
B. PERIODS FROM PHASE PLANE	94
REFERENCES	97
BIOGRAPHICAL SKETCH	101

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This study investigates the behavior of simple bouncing systems, namely simple oscillators which are released from a certain height. In particular, a nonlinearity exists in the discontinuity of two different dynamic system modes, which are the flight mode and the ground mode, although the behavior of such a system is linear in each mode. Such oscillators provide models for mechanical systems such as legged systems for hopping, in which components make intermittent contact.

The phase plane technique and the power spectrum analysis, which provide simple yet powerful tools of the dynamic analysis for linear and nonlinear systems, permit not only the determination of parameters for the stability of bilinear bouncing systems, but also the investigation of the

chaos that may occur. It is necessary to examine what causes the chaotic behavior of the simple bouncing systems in order to design a practical leg system.

From the analyses of the linear single degree of freedom and the linear two degree of freedom oscillatory systems, some of the chaotic responses at the critical frequencies for the bilinear two degree of freedom bouncing systems can be predicted.

CHAPTER 1 INTRODUCTION

1.1 Problem Statement and Previous Work

The motivation for this study came originally from the area of legged locomotion. Legged locomotion can be attained by a motion that combines a vertical bouncing of the body with a fore-aft swinging of the leg. Legs are the elements that exert forces on the body to propel the body forward for transport, and to keep the body in an upright posture. Spring and damping elements in the leg systems can reduce maximum loading and simplify control.

In this research, leg systems for hopping are considered because of the interest in emulating human or animal locomotion as a biped or a quadruped. Central to the design of a legged machine is the mechanical design of the leg itself. Legged systems should be able to generate bouncing motions of the body and swinging motions of the leg for transport. In particular, repeated bouncing motions as a part of legged locomotions are investigated to design a leg. Chaotic behaviors of the simplified bouncing system are found to exist depending on the system parameters and initial conditions.

There has been much research on the subject of legged locomotion. Those may be divided into two major fields: (dynamic) modelling and control of a legged locomotion, and construction of experimental legged vehicles.

The present major obstacle for development of highly mobile and practical legged vehicles stems from the lack of a solid mathematical description of legged locomotion. As a result, a deficiency exists in mastering the associated control problems. Lagrangian dynamics and modern control theory have been applied to the study of legged locomotion, especially biped [1-12]. Although these techniques have been successful in providing an understanding of the unstable dynamics associated with bipedal postural stance and optimal trajectories for bipedal systems as coupled rigid body systems, the models which were used for these studies are far from realistic for human or animal locomotion systems. For any given motion, certain degrees of freedom are dominant while others can be neglected. Hence, to study the dynamics of legged locomotion systems, it becomes imperative to select mechanical models having few degrees of freedom to keep the equations of motion to a manageable level, yet having enough degrees of freedom to represent the motions involved.

Compound inverted pendulums with no more than three degrees of freedom have been employed to study the dynamics of postural stance as mechanical models of a biped. Hemami et al. [3-9] examined periodic motion generation, trajectory stabilization, and trajectory transition controls for

inverted pendulums. Since controls for practical bipeds should be capable of producing stable locomotion gaits, and of providing transitions from one periodic motion to another, the results for the inverted pendulums may be useful for further development of robot locomotion controls.

McMahon and Mochon [11, 12] described the swing phase of human gait as a ballistic motion of a pure pendulum. They assume that the muscles act only to establish an initial configuration and velocity of links at the beginning of the swing phase. The swinging leg and the rest of the body then moves through the remainder of the swing phase entirely under the action of gravity. The computed range of times of swing for the model was found to be very close to experimental results.

However, neither the inverted pendulum nor the pure pendulum alone can be used to model complete cycles of walking or running. While one leg supports the body like an inverted pendulum, another leg is in the swing phase at the same time in bipedal locomotion. Thus, both the inverted pendulum and pure pendulum should be combined to achieve the bipedal locomotion cycles.

Seireg and Townsend [1] presented a decomposition scheme which separates the dynamics and the kinematics, and a numerical algorithm which provides time optimal control functions for the nonlinear dynamic problems of systems of coupled rigid bodies with application to a bipedal locomotion system. The time-optimal control functions and trajectories

may be used for the synthesis of closed-loop controls for the system [2]. Although the methods demonstrated the existence of a solution to given systems of coupled rigid bodies, the uniqueness cannot be guaranteed.

A historical review of research on legged machines can be found in Raibert [13] and Todd [14]. However, some of the outstanding works will be described here briefly. In the late 1960's and the 1970's some work on exoskeletons was done by Grundman and Seireg [15]. A series of hydraulic exoskeletons for paraplegics was built, which were programmed for standing up, sitting down, and stair climbing. A fully computer controlled quadruped walking machine was built by Frank and McGhee at the University of Southern California [14, 16]. Each leg possessed two degrees freedom, and the joint coordination was performed by a computer. In the 1970's, Ohio State University (OSU) started a series of investigations on legged locomotion. The OSU Hexapod was built by McGhee and his associates [14]. This machine was fully controlled by a PDP 11/70 computer via an umbilical cord and was powered externally through a cable. Each leg had three degrees of freedom and was made of two links connected by joints. The group at OSU is currently developing a much larger hexapod (Adaptive Suspension Vehicle). The Adaptive Suspension Vehicle is intended to operate on rough terrain with a high degree of autonomy [17]. Other experimental walking machines include TITAN III, PV II, and ODEX I [14]. Even though these research efforts for

statically stable multi-legged vehicles have generated good results, few of them contribute to dynamically stable legged locomotion.

Miura and Shimoyama built the BIPER series [18]. The BIPER3 has knee joints but no ankle joint assuming a point contact between the foot and the ground. A continuous walking gait is required to prevent the BIPER3 from falling. The BIPER4 has a shape similar to human legs. Both are designed to walk following a preplanned trajectory.

Frusho and Masubuchi developed a hierarchical control structure [19]. In the lower level control, a local feedback at each joint is used, while in the upper level the reference inputs to the local feedback are corrected by using a reduced order model.

Zheng and Shen proposed a control scheme, using position and force sensors [20], which enables a biped to walk from a level surface to a slope. For a statically stable biped locomotion, the projection of the center of mass remains within the supporting area by moving the main body of the robot back and forth. The experimental biped robot, which has nine links and eight joints without knee joints, was built to verify the proposed control scheme.

For legged locomotions, some means for balance must be provided, since the body's center of mass is above the ground. The results of biped research have been limited to only slow walking gaits so that the dynamic effects of the system can be neglected and the static balance for the system

can be maintained. However, because of the need to maintain stability at all times, statically balanced locomotion is limited in speed and maneuverability. Recently, dynamic balance has been used to overcome these limitations. Dynamic balance requires attention not only to position and forces, but also to temporal aspects of limb control. For example, if a biped does not put its foot down in the right place at the right time, it falls down.

The first machines that balance actively were automatically controlled inverted pendulums. Cannon and Higdon [21] used a cart, on which one or more simple inverted pendulums were mounted, as the physical model. The pendulums were to be balanced by a controller which moves the cart back and forth. Using analysis based on bang-bang control logic, the regions of controllability were expressed as explicit functions of the physical parameters of the system. This study of balance for inverted pendulums was an important precursor to later work on locomotion. The inverted pendulum model for walking became the primary tool for studying balance in legged systems as discussed before. In 1983, a hopping machine was built by Raibert [13] at Carnegie-Mellon University. This hopping machine has only one leg, and must hop continuously to maintain balance. The leg has three degrees of freedom. The vertical motion is provided by a pneumatic cylinder which is mounted on the body frame via a gimbaled joint. Two hydraulic actuators control the lateral motion of the leg. This important piece of research has

provided considerable insight into the balancing problems of walking machines, and has been extended to a four legged bouncing machine. A variety of control procedures were used for a steady state motion which repeats itself after each hop. In fact, most research on legged locomotion concentrated mainly on controls for stability.

Numerous studies of legged locomotion deal with regular periodic running or walking. However, in order to design a leg system for a practical legged vehicle, the specific nonlinear dynamics should be analyzed, since nonlinearities could produce nonperiodic or chaotic motions [22, 23]. Nonlinearity exists between the flight mode and the ground mode of the system when bouncing motion is considered as a part of legged locomotions.

1.2 Chaotic Systems

Recently, new phenomena have been observed in all areas of nonlinear dynamics. Chaotic oscillations, which seem to be random, are motions from completely deterministic systems. Such motions had been known in fluid mechanics, but they have been explored in low-order mechanical and electrical systems and even in simple one degree of freedom systems. A certain level of mathematical understanding has to be acquired in order to study chaotic phenomena.

The study of chaotic vibrations is important to engineering vibrations for several reasons. First, in

mechanical systems, a chaotic system makes life prediction or fatigue analysis difficult because the precise history of the system is not known. Second, the recognition that simple nonlinearities can lead to chaos raises the question of predictability in classical physics. For nonlinear systems with chaotic dynamics, the time history is sensitive to initial conditions and definite knowledge of the future may not be possible even when the motion is periodic.

Thompson et al. [32] defined chaos in the negative as recurrent behavior that is not an equilibrium, a cycle or even a quasi-periodic motion. Chaotic motion has some random aspects. The randomness arises from sensitive dependence on initial conditions, resulting for example in broadband noise in the power spectra of observable time histories. This seems remarkable because the dynamic systems need no stochastic input to achieve this. Even more surprising is that chaotic motions can be observed in quite simple dynamic systems [32]. It is necessary to distinguish between random and chaotic motions. The former one is for problems where the input forces are totally unknown or only some statistical measures of the parameters are given. The term chaotic is for those deterministic problems for which there are no random or unpredictable inputs or parameters. In fact, Thompson et al. [32] replaced the negative definition of chaos by a more positive one: chaos is recurrent motion with some random aspect in nonlinear dynamic systems. Exponential

divergence from adjacent starts while remaining in a bounded region of phase space is a signature of chaotic motion [32].

Chaotic vibrations may occur when nonlinearity exists. Examples of nonlinearities in mechanical systems include nonlinear elastic or spring elements, nonlinear damping, and backlash. To identify nonperiodic or chaotic motions, the following tests can be performed:

- a) Identify nonlinear elements in the system.
- b) Check for sources of random input in the system.
- c) Observe time history.
- d) Look at phase plane trajectories.
- e) Examine Fourier spectrum of signal.

When the motion is periodic, the phase plane trajectory is a closed curve. For example, the phase plane trajectory of a linear spring-mass system exhibits an ellipse. However, a nonlinear system may show an orbit that crosses itself but is still closed. This can represent a subharmonic oscillation. Systems for which the force does not depend explicitly on time are called autonomous. For autonomous nonlinear systems without harmonic inputs, periodic motions are referred to as limit cycles and are closed in the phase plane. Chaotic motions exhibit different behavior. Their phase plane trajectories are never closed or repeated. Thus, the trajectory will tend to fill up a section of the phase space. This is a strong indication of chaos.

One of the methods for detecting chaotic vibrations is the presence of a broad spectrum of frequencies. This

characteristic of chaos is very important for the low dimensional system. Often, if there is an initial dominant frequency, a precursor to chaos is the appearance of subharmonics in the frequency spectrum. One must be cautioned against concluding that multiharmonic outputs imply chaotic vibrations since the system in question might have many hidden degrees of freedom.

For chaotic vibrations, one or more of the system parameters must be varied to see if the system has steady or periodic behavior for some range of the parameter. In this way, it can be decided if the system is in fact chaotic and if there are no hidden inputs or sources of truly random noise. In changing a parameter, a pattern of periodic responses are searched for. One characteristic clue to chaotic motion is the existence of subharmonic periodic vibrations.

Although chaotic phenomena have been observed in many physical systems, chaotic systems which are closely related to bouncing motions will be listed here.

Impact-type problems have emerged as an obvious class of mechanical examples of chaos. Lichtenberg and Lieberman [24] used a model for the motion of a particle between two walls, where one wall is stationary and the other is oscillatory. Numerical studies of this model reveal that stochastic-type solutions exist so that most of the phase space is filled up. This model is similar to a bilinear oscillator whose mass slides freely on a shaft with viscous damping until it hits

stiff springs on either side [25, 26]. Another impact-type mathematical model is a bouncing ball on a table which is investigated by Holmes [27]. Experiments on the chaotic bouncing ball have been performed by Tufillaro and Albano [28].

A practical realization of impact-induced chaotic vibrations is the impact print hammer experiment by Tung and Shaw [29]. In this printing device, a hammer head is accelerated by magnetic force and the kinetic energy is absorbed in pushing ink from a ribbon onto paper. When the print hammer is excited by a periodic voltage, it will respond periodically as long as the frequency is low. However, as the frequency is increased, the hammer has little time to damp or settle out and the impact history becomes chaotic. Thus, chaotic vibrations restrict the speed at which the printer can work.

Compliant offshore structures and articulated mooring towers which have been modelled by a bilinear oscillator have been studied by Thompson et al. [30-32]. A bilinear oscillator with different stiffness for positive and negative deflections arises frequently in off-shore marine technology due to the slackening of mooring lines. When one of the stiffness becomes infinite, the system becomes an impact oscillator. Harmonic, subharmonics, and chaotic motions were found to exist for this model.

Numerical simulations have been carried out to study chaotic phenomena. The time history of chaotic systems is

sensitive to slight changes in initial conditions and system parameters. This sensitive dependence of chaotic systems also raises questions about the accuracy of predicting time response by numerical simulations. Toungue [33] and Koh et al. [34, 35] discussed the effects of time step size on the numerical solutions of chaos.

In view of the great variety of behavior observed for nonlinear systems, it would be impossible at this time to make sweeping generalizations about routes to chaos. The question of which types of response may be found, and under what conditions they are stable, especially in relation to the choice of initial conditions, is extremely difficult, and it cannot be answered with full rigor.

1.3 The Goal and Organization of the Work

The goal of this research is to understand chaotic bouncing motions in order to design a practical leg system for bouncing. Since bouncing systems may exhibit chaos, it is desirable to predict system behavior with selected system parameters.

In the following chapter, the phase planes of various simplified bouncing systems will be displayed. Chaotic bouncing motions are found to exist for two degree of freedom bouncing systems depending on system parameters and initial conditions. In Chapter 3, spectrum analyses for chaotic bouncing systems will be described in detail. A separation

method is used to obtain dominant frequencies for each dynamic mode of the system. Frequency relationships are found after examining the results of spectrum analyses. They can be used to predict the system behavior with selected system parameters and given initial conditions. Chapter 4 explains elimination methods for chaotic bouncing motions.

CHAPTER 2 SIMPLIFIED BOUNCING SYSTEMS

2.1 Background

Running is a motion that combines a vertical bouncing motion with a swing motion of the leg. Legged systems for running should be able to generate a lift and a thrust by the leg simultaneously. In his pioneering work, Raibert has examined theoretical and experimental models of one-legged and multi-legged hopping robots [13]. One of the major functions of his hopping machines is the bouncing motion by the telescopic pneumatic leg system. Since point contact between the tip of the leg and the ground was assumed, i.e., without consideration of a foot, the one-legged hopping robot is a simple one degree of spring-mass bouncing system [22, 23]. The mass represents the body while the spring itself is a springy leg. However, for a smooth motion, a foot with an ankle joint would be required. A legged system with a foot (without an ankle) for the bouncing motions can be modelled as a two degree of freedom spring-mass system. One mass is for the body, and another mass is for the foot. A leg is modelled as a spring between the two masses, and the compliance of the foot is modelled by a second spring. It is assumed that there is no energy dissipation for the

simplified models.

2.2 One Degree of Freedom Spring-Mass System

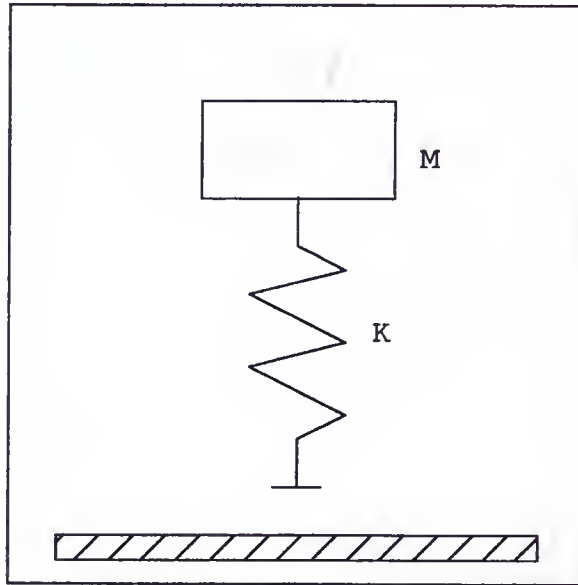


Figure 2.1 One Degree of Freedom Bouncing System

Consider the one degree of freedom spring-mass system which is dropped from a certain height (h) and bounces back (Figure 2.1). This represents the simplest model for the bouncing motion of a legged system. The mass (M) and the spring (K) denote the body and the leg, respectively. When the system hits the ground, the potential energy of the system is converted to the elastic strain energy in the compressed spring. This stored energy will be used for the rebound when the spring loses contact with the ground.

This model has two different modes. One is a linear oscillatory mode (Ground Mode) when the spring maintains contact with the ground, and the other is a free fall mode in a gravitational field (Flight Mode) when the spring is not in

contact with the ground. A nonlinearity occurs due to the discontinuity of two different dynamic system modes, although the behavior of such a system is linear in each mode. The dynamic equations of the system for each mode are:

$$\text{(Ground Mode)} \quad M\ddot{x} + Kx = -Mg \quad (2.1)$$

$$\text{(Flight Mode)} \quad M\ddot{x} = -Mg \quad (2.2)$$

where M is the mass, K is the stiffness, and g is the gravity.

The displacement x is measured from the position of the body M at the instant when the spring first reaches the surface. The position x is negative if the body is below this reference point. Thus, if x is greater than zero, the system is in the flight mode while the system is in the ground mode with x less than or equal to zero.

2.3 Phase Planes of One Degree of Freedom Bouncing Systems

The phase planes of one degree of freedom spring-mass bouncing systems are all obtained for a fixed body mass of M ($= 10 \text{ Kg}$). Since the displacement x is measured from the position of the body M at the instant when the spring first touches the ground, $x = 0$ and $v = -\sqrt{2gh}$ at $t = 0$ are used as initial conditions for the bouncing motions.

Six different cases were selected to simulate bouncing motions of one degree of freedom bouncing systems by Runge-Kutta method with 0.001 second as a time step.

Cases of Dropping Height (h) = 0.2 m

When the dropping height is 0.2 m, i.e., the system is raised 0.2 m and is dropped from that height, $K = 575 \text{ N/m}$ (Figure 2.2.a) and $K = 1900 \text{ N/m}$ (Figure 2.2.b) are used as examples of one degree of freedom systems.

Cases of Dropping Height (h) = 0.5 m

With $h = 0.5 \text{ m}$, $K = 1860 \text{ N/m}$ (Figure 2.3.a) and $K = 3080 \text{ N/m}$ (Figure 2.3.b) are applied as system parameters.

Cases of Dropping Height (h) = 1.0 m

With $h = 1.0 \text{ m}$, $K = 430 \text{ N/m}$ (Figure 2.4.a) and $K = 2010 \text{ N/m}$ (Figure 2.4.b) are assigned to examine bouncing motions.

For all cases of single degree freedom systems, stable bouncing motions were demonstrated with phase plane plots. In the upper half of the phase plane which is the flight mode, displacement has a parabolic relationship with velocity. For the ground mode, displacement and velocity display the shape of an ellipse. If the velocity divided by the natural frequency of the system in the ground mode is used for the horizontal axis of the phase plane, then the phase plane trajectory for the ground mode is a circle instead of an ellipse.

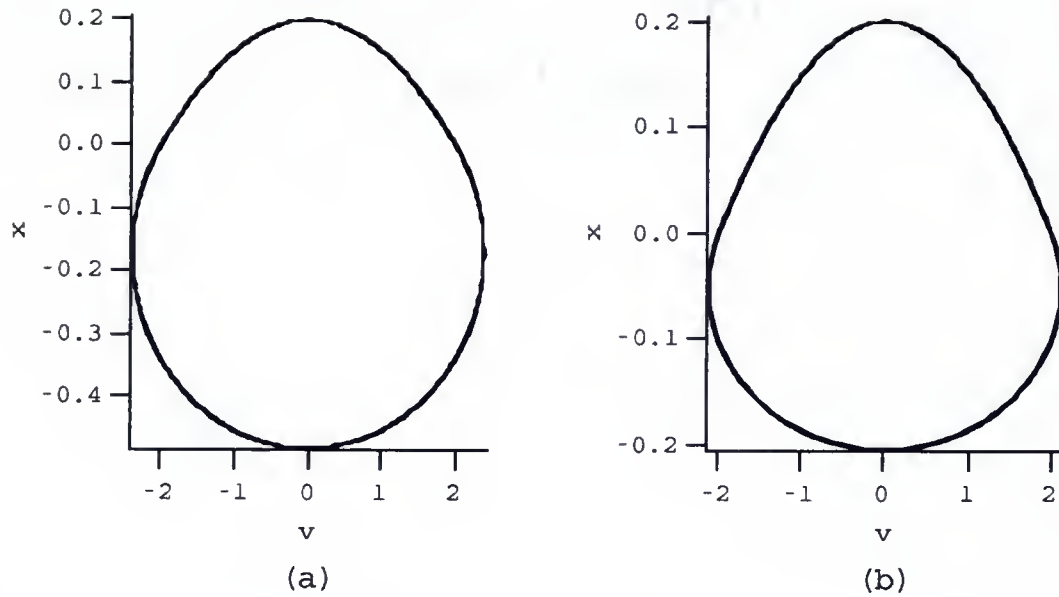


Figure 2.2 One Degree of Freedom Bouncing
with $h = 0.2 \text{ m}$, $M = 10 \text{ Kg}$
(a) $K = 575 \text{ N/m}$, (b) $K = 1990 \text{ N/m}$

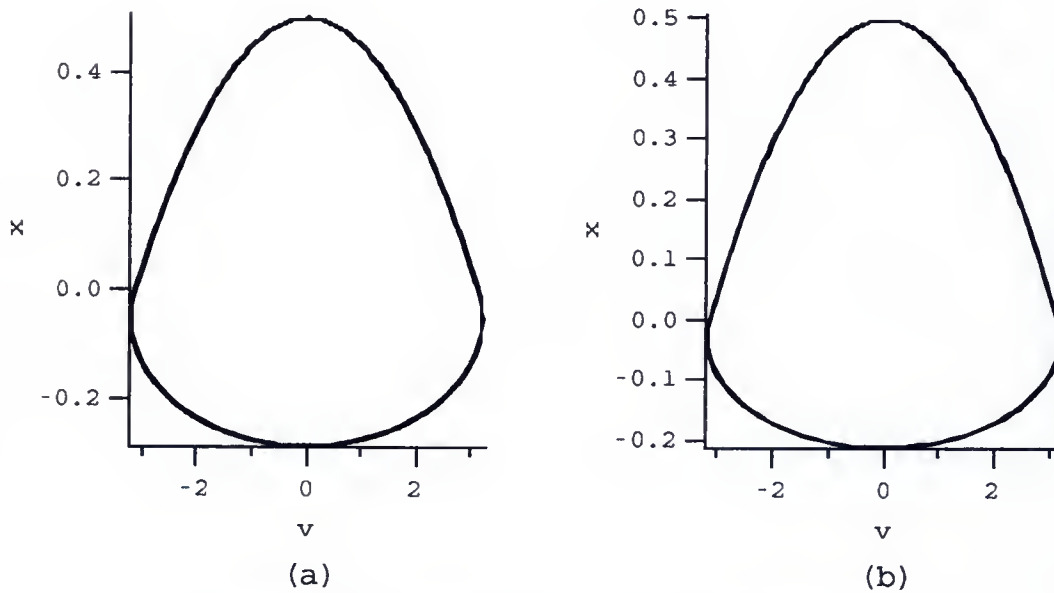
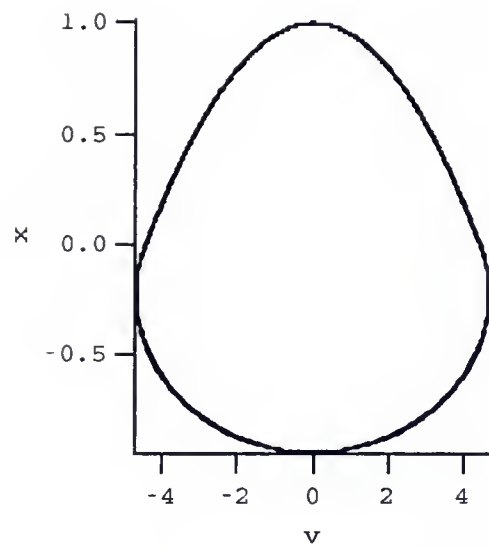
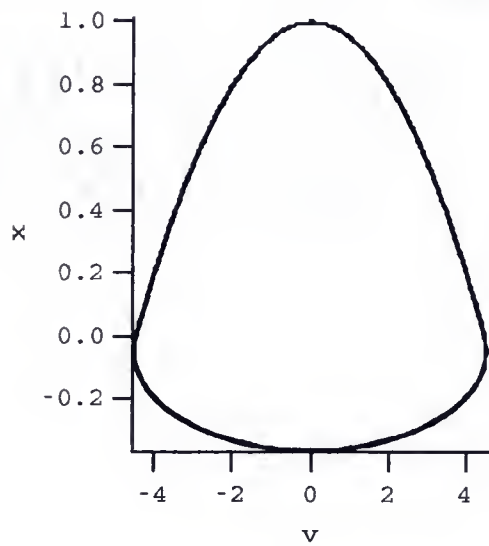


Figure 2.3 One Degree of Freedom Bouncing
with $h = 0.5 \text{ m}$ and $M = 10 \text{ Kg}$
(a) $K = 1860 \text{ N/m}$, (b) $K = 3080 \text{ N/m}$



(a)



(b)

Figure 2.4 One Degree of Freedom Bouncing
with $h = 1.0\text{m}$ and $M = 10\text{ Kg}$
(a) $K = 430\text{ N/m}$, (b) $K = 2010\text{ N/m}$

2.4 Two Degree of Freedom Spring-Mass Systems

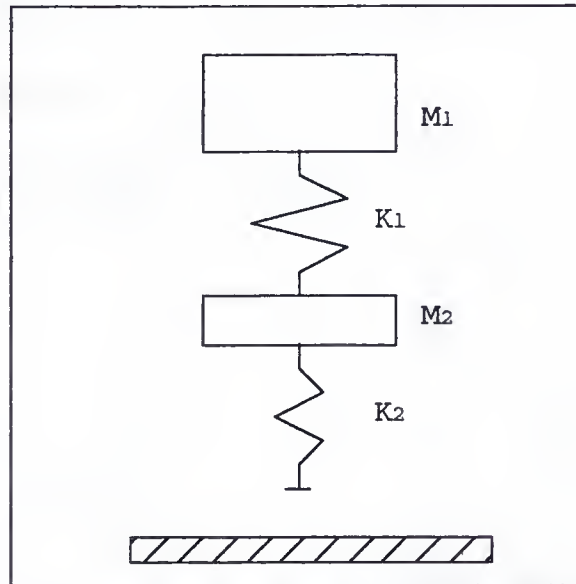


Figure 2.5 Two Degree of Freedom Bouncing System

For Raibert's hopping machines, a point contact between the end of the leg and the surface with enough frictional forces between them to prevent slipping, was assumed for simplicity. However, for a smooth motion as seen in animal locomotion, a foot is also required. Another spring-mass system as a foot is added to the one degree of freedom spring-mass system. This two degree of freedom spring-mass bouncing system also has two different modes, the flight mode and the ground mode. For the ground mode, it is a simple linear oscillatory system with two masses and two springs, and the motion of the system for the flight mode is a combination of a free fall in a gravitational field and oscillations between the two masses.

The equations of motion are:

$$\text{(Ground Mode)} \quad M_1 \ddot{x}_1 + K_1 x_1 - K_1 x_2 = -M_1 g \quad (2.3)$$

$$M_2 \ddot{x}_2 - K_1 x_1 + (K_1 + K_2) x_2 = -M_2 g \quad (2.4)$$

$$\text{(Flight Mode)} \quad M_1 \ddot{x}_1 + K_1 x_1 - K_1 x_2 = -M_1 g \quad (2.5)$$

$$M_2 \ddot{x}_2 - K_1 x_1 + K_1 x_2 = -M_2 g \quad (2.6)$$

where M_1 and M_2 are the body and the foot mass, K_1 and K_2 are the leg and the foot stiffness, and g is gravity.

The displacements (x_1 and x_2) are measured from the positions of the bodies (M_1 and M_2) at the instant when the foot spring (K_2) contacts the floor for the first time. If the system is initially raised to a certain height (h) and dropped from that height, it is assumed that there are no interactions between the two bodies until the foot spring touches the surface at least once, i.e., the starting free fall motion of the system is a rigid body motion so that initial conditions for the two different bodies are the same. The condition for the ground mode is that x_2 is less than or equal to zero. As soon as the foot spring (K_2) hits the surface, the system begins the ground mode since the displacement of the foot (M_2) becomes zero and negative thereafter. The system is in the flight mode when the displacement of the foot (x_2) is greater than zero. In other words, it is the ground contact reference point when x_2 is equal to zero. The conditions for the ground mode and the flight mode are independent of the body displacement (x_1).

2.5 Phase Planes of Two Degree of Freedom Bouncing Systems

For two degree of freedom spring-mass bouncing systems, the body mass (M_1) = 10Kg, the foot mass (M_2) = 1Kg, and the leg stiffness (K_1) = 1000N/m were used as the fixed system parameters. The fixed parameters for the body and the foot were selected in order to emulate animal mass ratio of body and foot. Since the system is initially raised to a certain height (h) and dropped from that height, $x_1 = x_2 = 0.0$ and $v_1 = v_2 = -\sqrt{2gh}$ at $t = 0$ are used as initial conditions for the bouncing motions where x_1 and v_1 , x_2 and v_2 are displacements and velocities of the body and the foot, respectively.

Six different cases were selected to simulate bouncing motions of two degree of freedom systems by Runge-Kutta method with 0.001 second as a time step¹.

Cases of Dropping Height (h) = 0.2m

When the dropping height is 0.2m, i.e., the system is initially raised to the height of 0.2m and is released from that height, $K_2 = 575\text{N/m}$ (Figure 2.6) and $K_2 = 1900\text{N/m}$ (Figure 2.7) are used as examples of two degree of freedom bouncing system parameters.

Cases of Dropping Height (h) = 0.5m

With $h = 0.5\text{m}$, $K_2 = 1860\text{N/m}$ (Figure 2.8) and $K_2 = 3080\text{N/m}$ (Figure 2.9) are applied as system parameters in order see how systems behave with changes of initial conditions and

¹ see Appendix A for the effects on time step size

system parameters.

Cases of Dropping Height (h) = 1.0m

With $h = 1.0\text{m}$, $K_2 = 430\text{N/m}$ (Figure 2.10) and $K_2 = 2010\text{N/m}$ (Figure 2.11) are assigned to examine the bouncing motions of the two degree of freedom systems with different initial conditions and system parameters.

The phase planes in Figure 2.7.a, Figure 2.9.a, and Figure 2.10.a for the body (M_1) have repeated bouncing motions which are similar to those of the single degree of freedom bouncing systems. However, according to Figure 2.6.a, Figure 2.8.a, and Figure 2.11.a, chaotic motions can be seen in the phase planes for the body. The dynamic responses of the body and the foot seem to have strong relationships. When the shape of the phase plane for the foot (M_2) is simple as in Figure 2.7.b, Figure 2.9.b, and Figure 2.10.b, the body has repeated bouncing motions which are desirable aspects for bouncing systems. On the other hand, the trajectories of the phase plane for the body have deviations with obvious chaotic motions of the foot (Figure 2.6.b, Figure 2.8.b, and Figure 2.11.b). By introducing another mass-spring as a foot and a foot spring to the single degree of freedom system, a two degree of freedom bouncing system may have chaotic behavior depending upon the initial conditions and the system parameters.

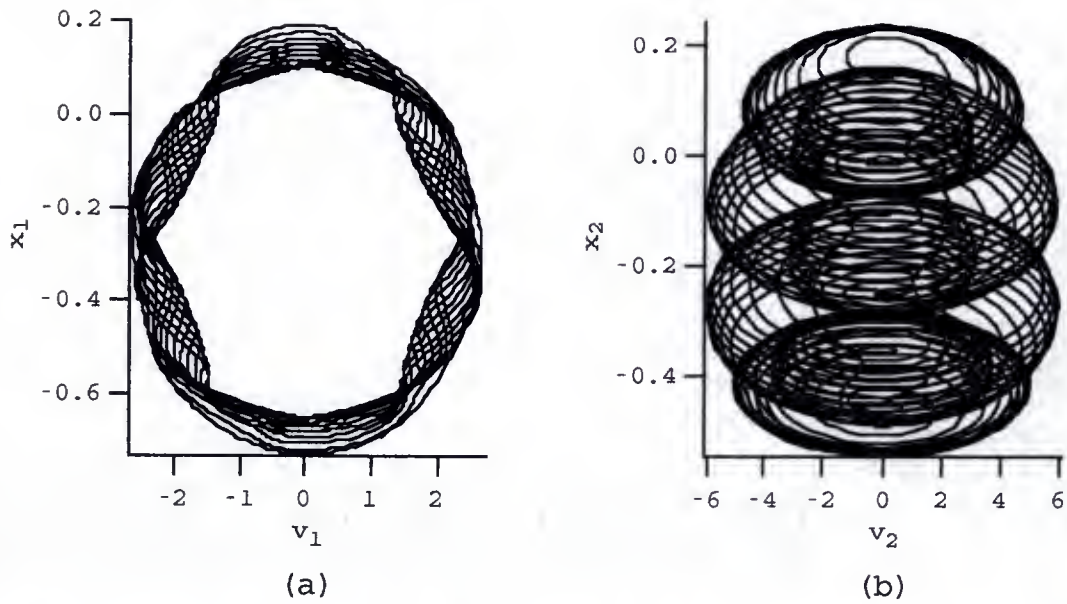


Figure 2.6 Phase Planes of Two Degree of Freedom System with $h = 0.2\text{m}$, $K_1 = 1000\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, and $K_2 = 575\text{N/m}$
 (a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

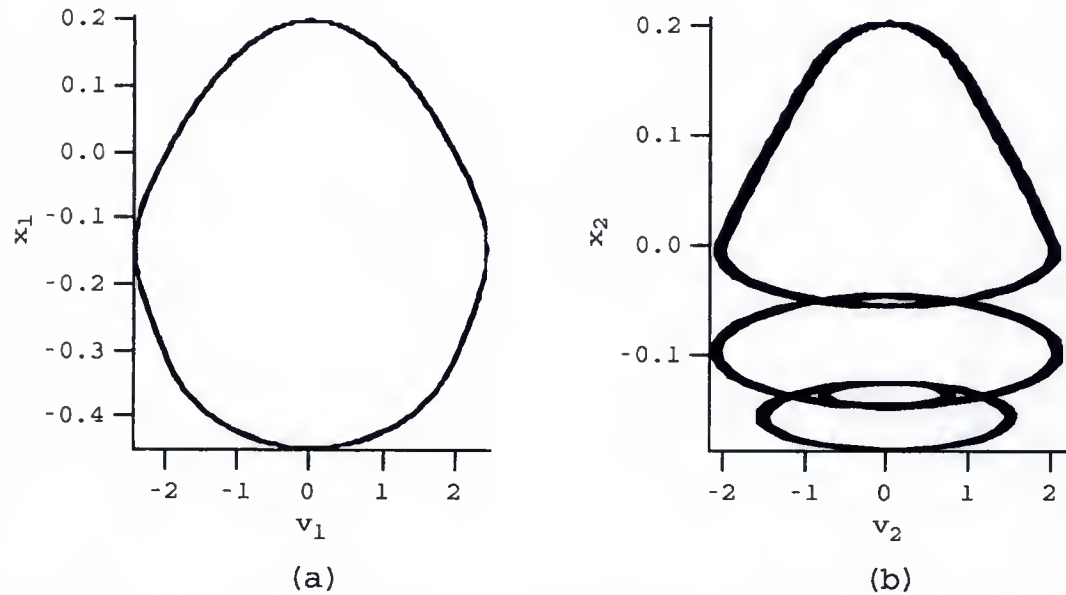


Figure 2.7 Phase Planes of Two Degree of Freedom System with $h = 0.2\text{m}$, $K_1 = 1000\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, and $K_2 = 1900\text{N/m}$
 (a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

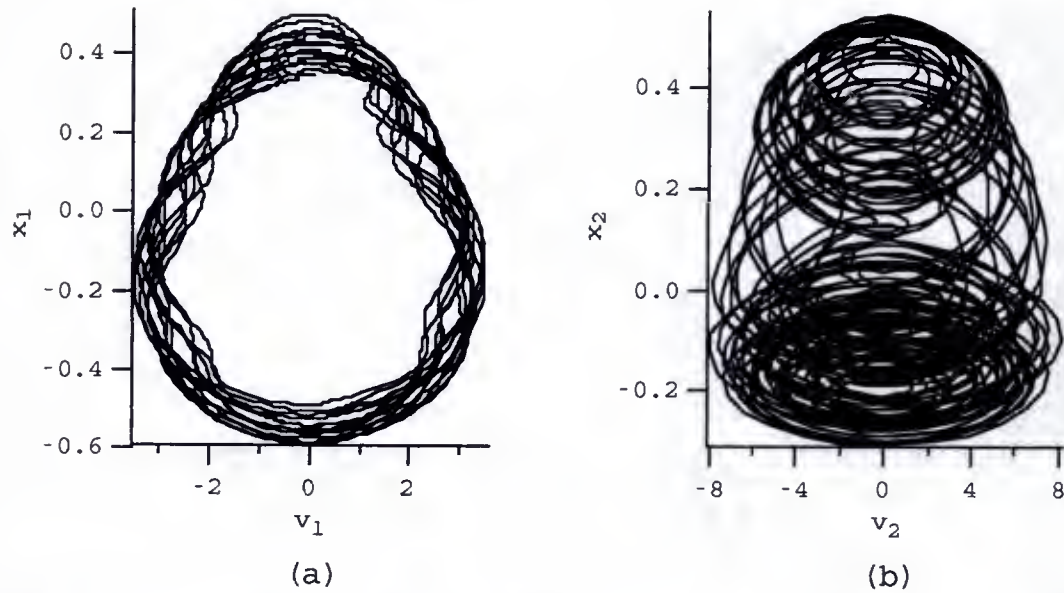


Figure 2.8 Phase Planes of Two Degree of Freedom System with $h = 0.5\text{m}$, $K_1 = 1000\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, and $K_2 = 1860\text{N/m}$
 (a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

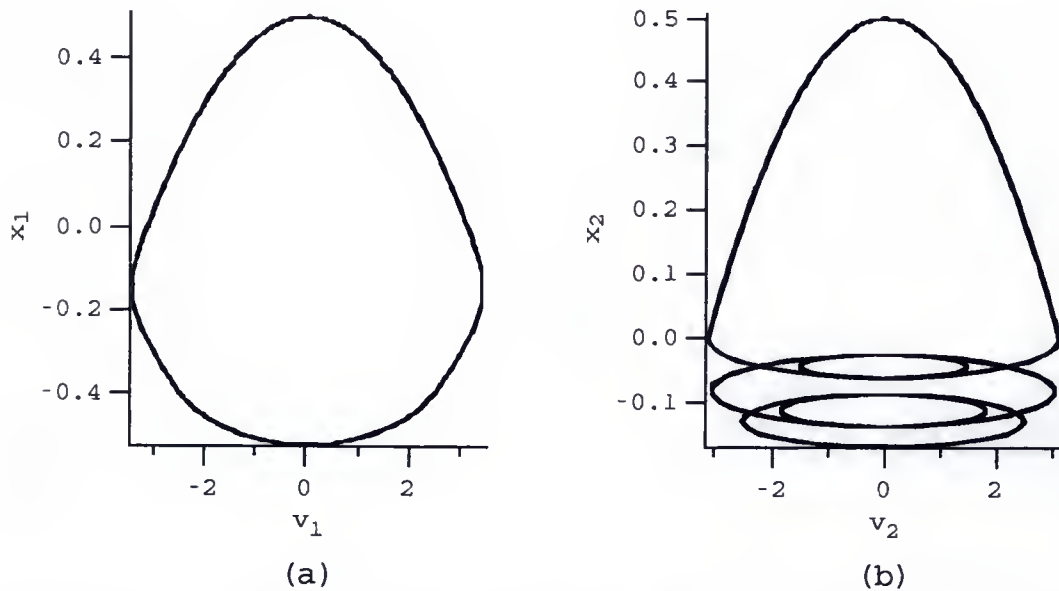


Figure 2.9 Phase Planes of Two Degree of Freedom System with $h = 0.5\text{m}$, $K_1 = 1000\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, and $K_2 = 3080\text{N/m}$
 (a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

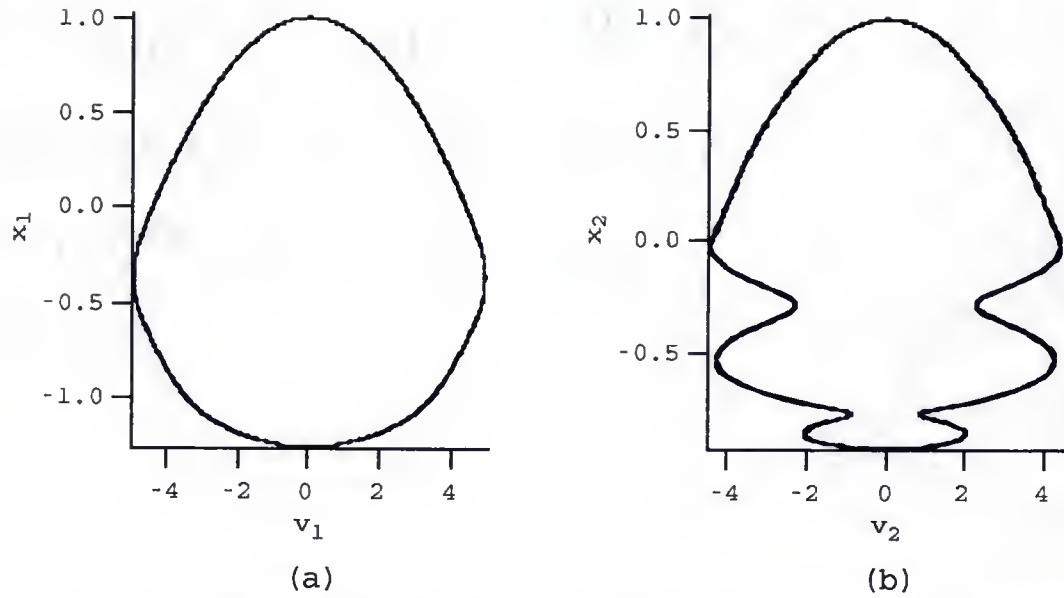


Figure 2.10 Phase Planes of Two Degree of Freedom System with $h = 1.0\text{m}$, $K_1 = 1000\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, and $K_2 = 430\text{N/m}$
 (a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

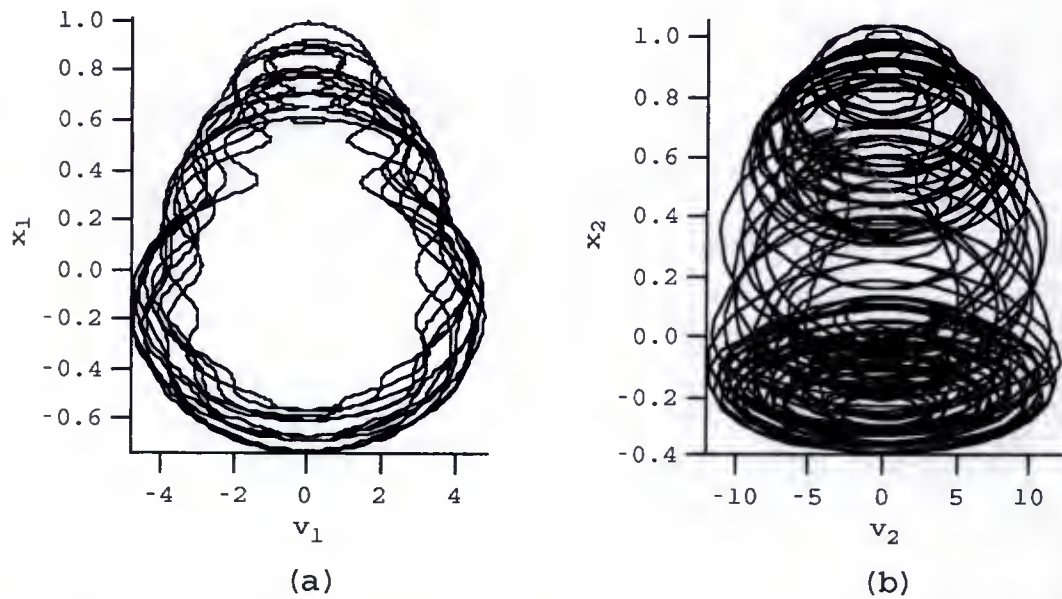


Figure 2.11 Phase Planes of Two Degree of Freedom System with $h = 1.0\text{m}$, $K_1 = 1000\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, and $K_2 = 2010\text{N/m}$
 (a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

CHAPTER 3
CHAOS IN TWO DEGREE OF FREEDOM BOUNCING SYSTEMS

3.1 Background

The chaotic behavior of two degree of freedom bouncing systems has been observed with the fixed system parameters for the body mass ($M_1 = 10\text{Kg}$), the leg stiffness ($K_1 = 1000\text{N/m}$), and the foot mass ($M_2 = 1\text{Kg}$), i.e., the foot stiffness (K_2) is the only system variable. These fixed system parameters will be used throughout the investigation of the chaotic system behavior.

The two degree of freedom bouncing system, which is linear in each mode, can be considered as three different systems depending on the dynamic modes of the system. The three different systems can be identified as the two degree of freedom linear oscillatory systems (Figure 3.1.a for the normal ground mode and Figure 3.1.b for the normal flight mode) along with the one degree of freedom linear oscillatory system (Figure 3.1.c for the rigid body ground mode). The first system for the ground mode (see Figure 3.1.a) is labelled *system a*, the second system for the flight mode (see Figure 3.1.b) is labelled *system b*, and the third system for the ground mode (see Figure 3.1.c) is labelled *system c*. When the bouncing system is represented by *system c*, it is

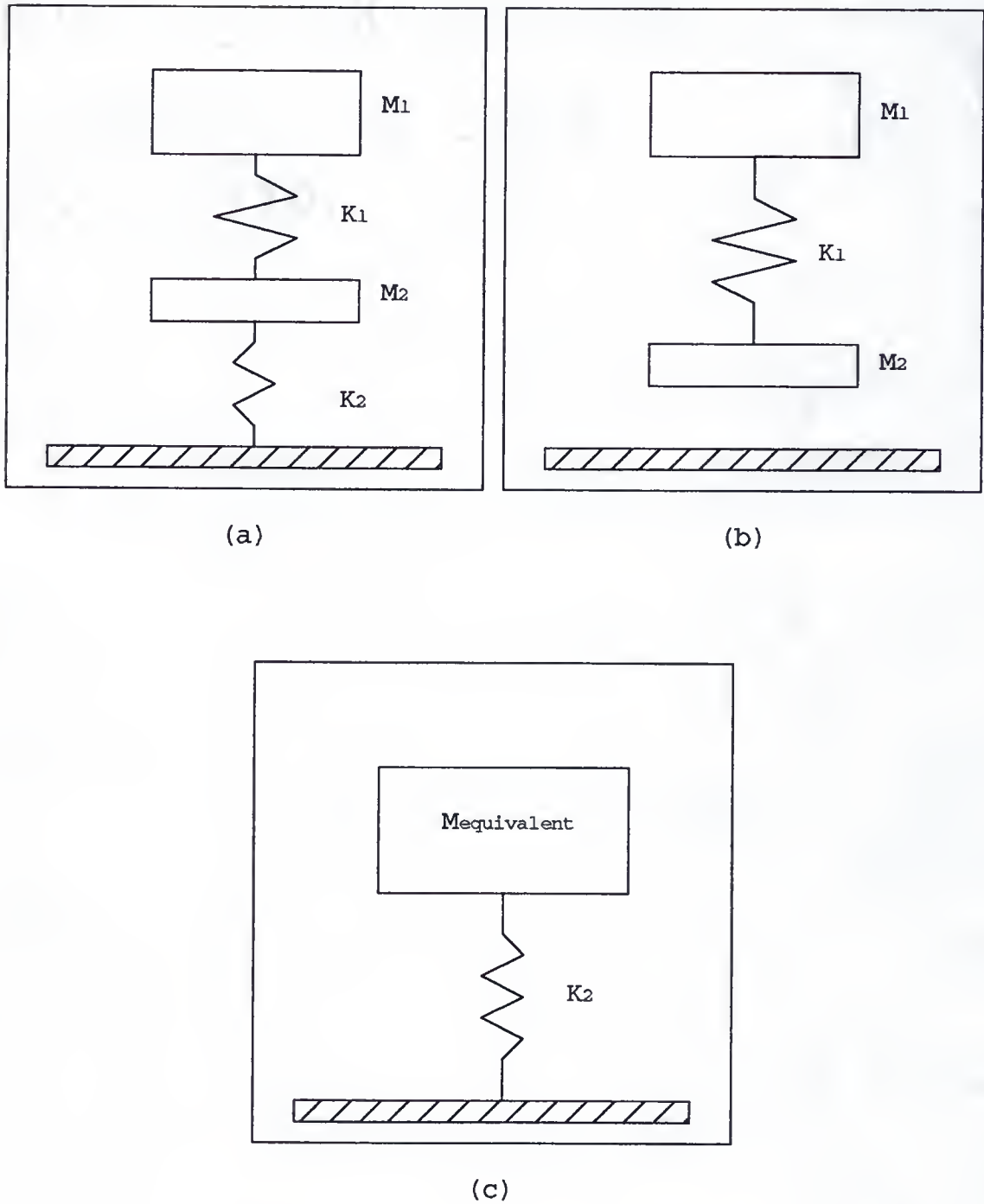


Figure 3.1 Three Different Linear Oscillatory Systems
 (a) *System a* for the normal ground mode
 (b) *System b* for the normal flight mode
 (c) *System c* for the rigid body ground mode
 ($M_{\text{equivalent}} = M_1 + M_2$)

assumed that there is no relative motion between the body mass and the foot (as in rigid body motion).

The natural frequencies (ω_{a1} and ω_{a2}) of *system a* are

$$\omega_{a1,a2} = \sqrt{\frac{K_1 M_1 + K_2 M_1 + K_1 M_2 \pm \sqrt{(K_1 M_1 + K_2 M_1 + K_1 M_2)^2 - 4 M_1 M_2 K_1 K_2}}{2 M_1 M_2}} \quad (3.1)$$

The natural frequencies (ω_{b1} and ω_{b2}) of *system b* are

$$\omega_{b1,b2} = 0, \sqrt{\frac{(M_1 + M_2) K_1}{M_1 M_2}} \quad (3.2)$$

Since ω_{b1} is equal to zero (i.e., the system is in rigid body mode) ω_b will be used as ω_{b2} for the natural frequency of *system b*.

The natural frequency of *system c* (ω_c) is

$$\omega_c = \sqrt{\frac{K_2}{M_{\text{equivalent}}}} \quad (3.3)$$

where M_1 is the body mass, M_2 is the foot mass, K_1 is the leg stiffness, K_2 is the foot stiffness, $M_{\text{equivalent}}$ is equivalent mass for the rigid body ground mode ($M_{\text{equivalent}} = M_1 + M_2$).

It should be noted that the natural frequency of *system c* (ω_c) is a convenience and does not correspond to any natural frequencies of the two degree of freedom system.

The objective of this chapter is to find the relationships between the dominant frequencies of the flight and the ground modes and the natural frequencies of the specified linear oscillatory systems. A Fast Fourier Transform (FFT) was used to obtain the dominating frequencies

of the flight and ground modes of the bouncing system.

3.2 Sum of Standard Deviations and Area in Phase Plane Plot for the Specified Region

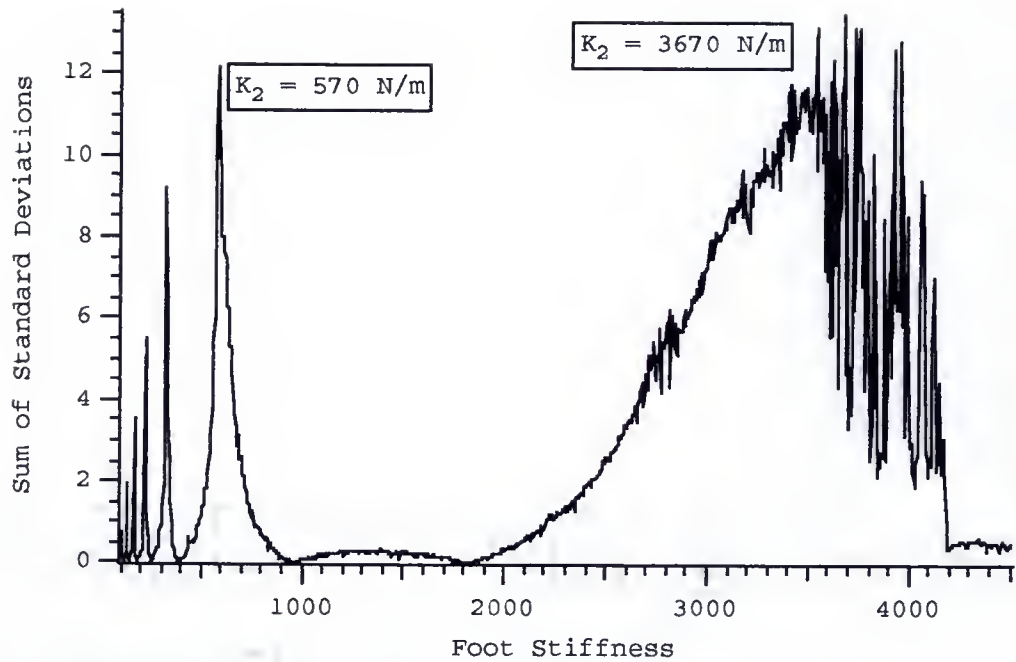
First, a statistical approach is used to select the foot stiffness which makes bouncing motions chaotic. Standard deviations for the data points of the vertical axis (x_1) with respect to each data point of a horizontal axis (v_1) in the body phase plane plot are obtained. These standard deviations are added together for the designated range of the horizontal axis. The range is set so that the data for the ground and the flight modes can be separated easily. For the cases of $h = 0.5\text{m}$ and $h = 1.0\text{m}$, the velocity (v_1) range between -2.0 and 2.0 is used. The horizontal axis range between -1.0 and 1.0 is used with $h = 0.2\text{m}$. The procedure to get the sum of the standard deviations is repeated for the minimum foot stiffness (K_2) = 100N/m and the maximum foot stiffness (K_2) = 4500N/m with an increment of 5N/m and with $h = 0.2\text{m}$ and 1.0m . However, the value of 4000N/m is used for the maximum foot stiffness (K_2) with $h = 0.5\text{m}$. The results are shown in Figure 3.2.a, Figure 3.3.a, and Figure 3.4.a. When the body phase plane trajectories are repeated, the sum of standard deviations is small. On the contrary, when the system with chosen foot stiffness has chaotic bouncing motions, then the sum of standard deviations is increased.

Second, the area of the body phase plane with the same velocity range is calculated to check whether it has

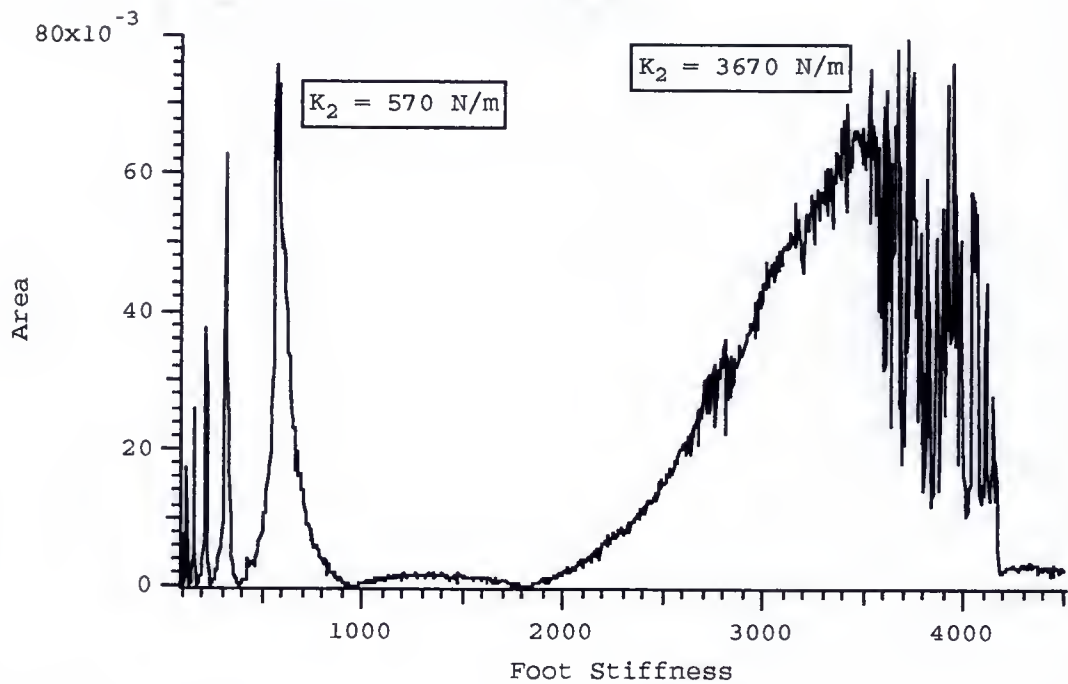
different criteria from those of the sum of the standard deviations. The area of the phase plane of the body for the given horizontal axis range versus foot stiffness is plotted in Figure 3.2.b, Figure 3.3.b, and Figure 3.4.b. These are almost identical to the results of the sum of standard deviations with respect to the foot stiffness.

The foot stiffness which makes the sum of standard deviations or area high is selected to obtain the dominant frequencies of the flight and ground modes.

In the following sections, spectrum analyses with selected system parameters will be discussed. For each case, the data for the regular two degree of freedom bouncing system will be initially used. The complete set of data will be separated into two sets of data as the flight mode and the ground mode to obtain the critical frequencies for each system mode. As a matter of fact, there are discontinuities between the end of the previous flight mode and the beginning of the the following flight mode, since there is a ground mode in the middle of two successive flight modes. However, it will be assumed that the end of the previous flight mode is followed by the beginning of the subsequent flight mode without any discontinuity. This assumption is also applied to the data of the ground mode. Then, the data sets with and without separations for the regular one degree of freedom bouncing system will be utilized. In this case, the sum of the body mass and the foot mass for the two degree of freedom system is the system mass ($M_{\text{equivalent}}$).



(a)

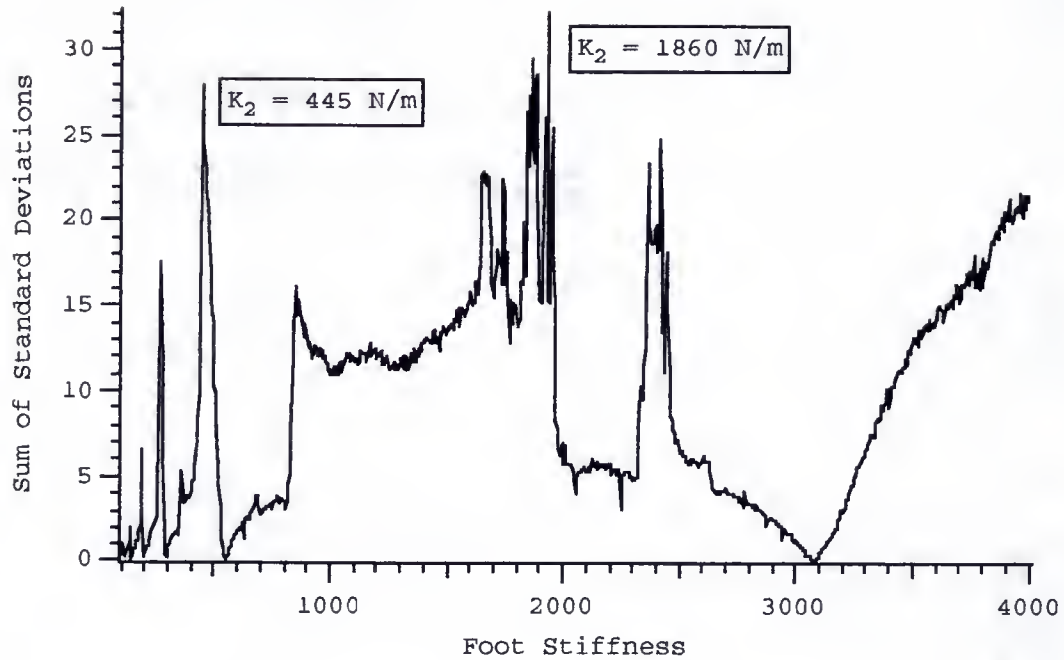


(b)

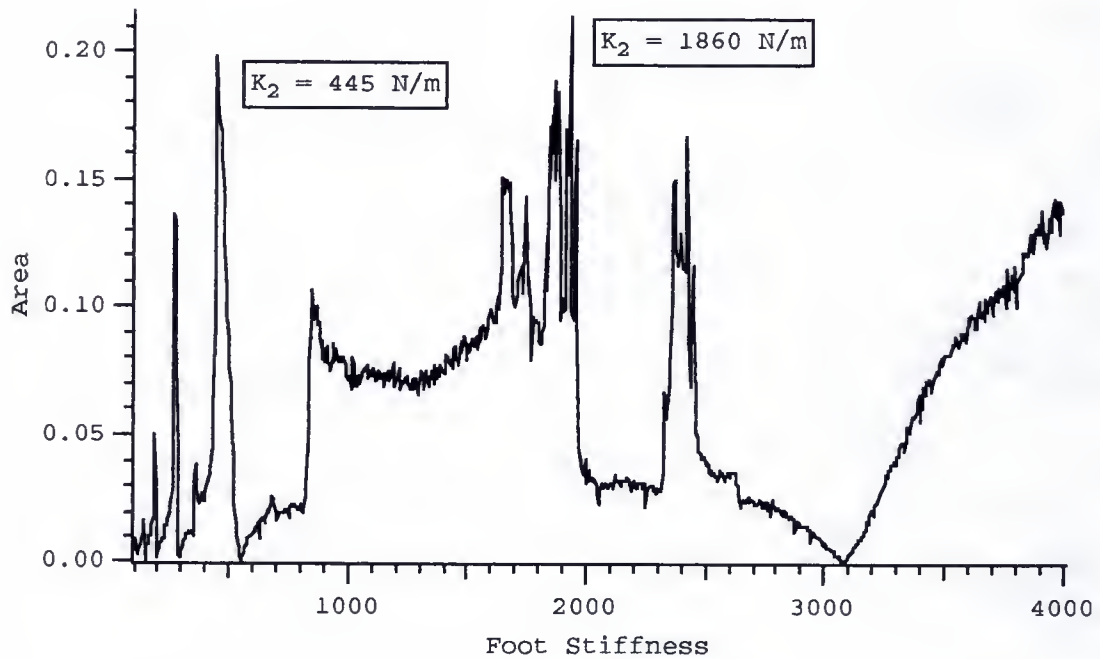
Figure 3.2 Sum of Standard Deviations and Area
 $(M_1 = 10\text{Kg}, M_2 = 1\text{Kg}, K_1 = 1000\text{N/m}, \text{ and } h = 0.2\text{m})$

(a) Sum of Standard Deviations

(b) Area



(a)

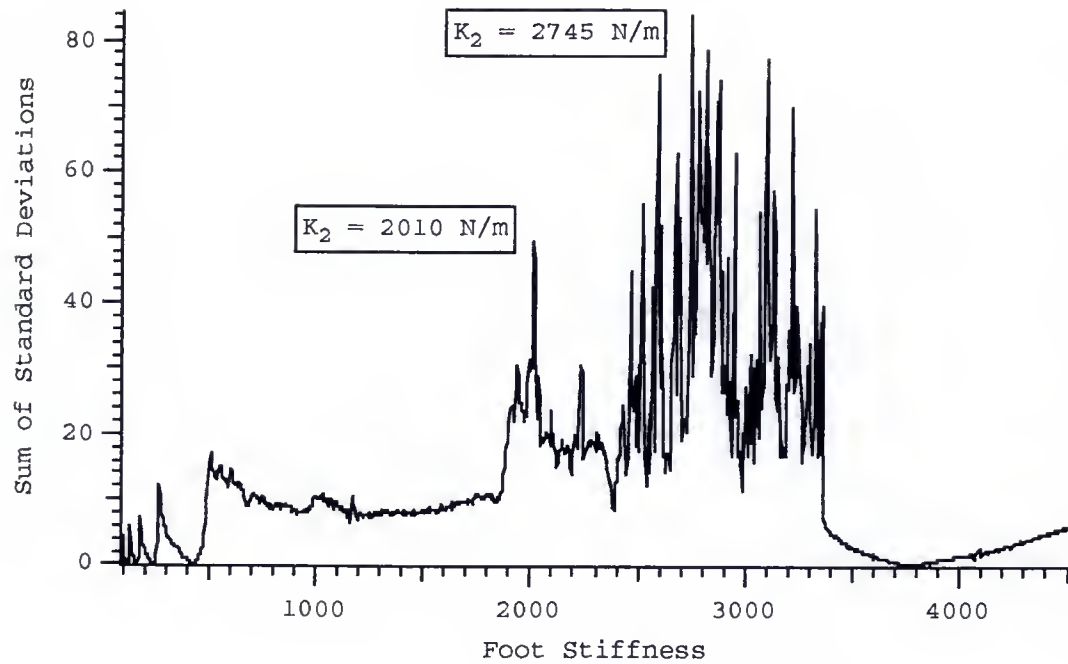


(b)

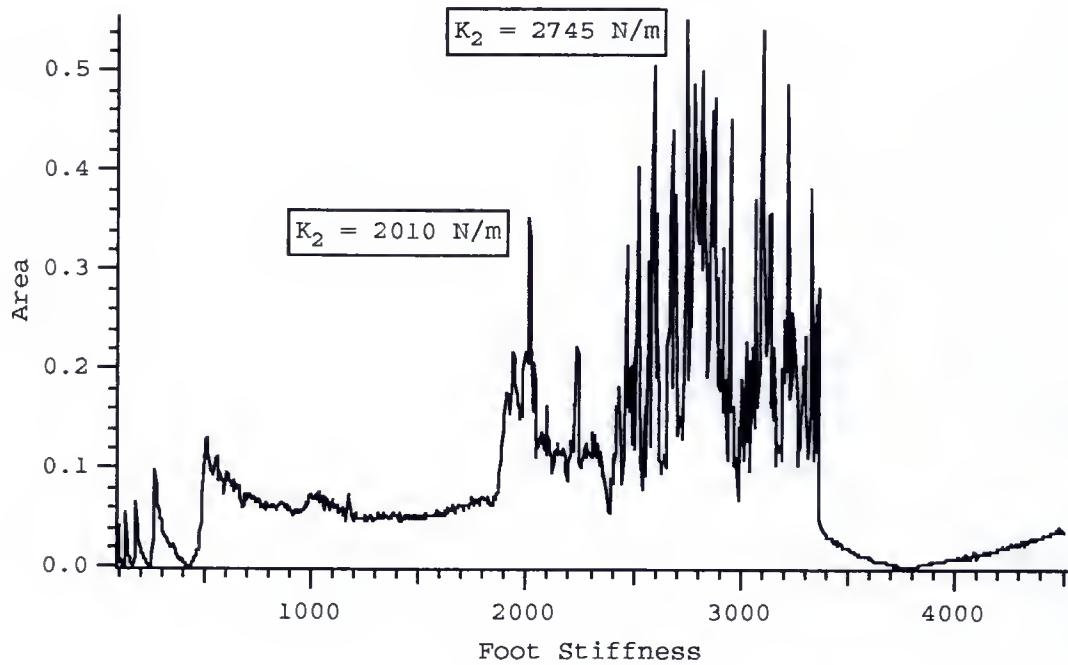
Figure 3.3 Sum of Standard Deviations and Area
 $(M_1 = 10\text{Kg}, M_2 = 1\text{Kg}, K_1 = 1000\text{N/m}, \text{ and } h = 0.5\text{m})$

(a) Sum of Standard Deviations

(b) Area



(a)



(b)

Figure 3.4 Sum of Standard Deviations and Area
 $(M_1 = 10\text{Kg}, M_2 = 1\text{Kg}, K_1 = 1000\text{N/m}, \text{ and } h = 1.0\text{m})$

(a) Sum of Standard Deviations

(b) Area

The period diagrams will also be used to study the chaotic system behavior. Since the period diagram is the intervals of the flight mode and the ground mode, the frequencies of each mode can be obtained easily for the one degree of freedom system. However, for the chaotic bouncing motions, sudden mode changes with a very short period can be observed with these period diagrams.

3.3 Spectrum Analyses for the Chaotic Bouncing Systems

with $h = 0.2\text{m}$

570N/m and 3670N/m for the foot stiffness (K_2) are selected as system parameters for the spectrum analyses of the chaotic bouncing systems with $h = 0.2\text{m}$. According to Figure 3.2, they correspond to the high values of the sum of standard deviations and area in the phase planes. The natural frequencies with the chosen system parameters for *system a*, *system b*, and *system c* are listed in Table 3.1.

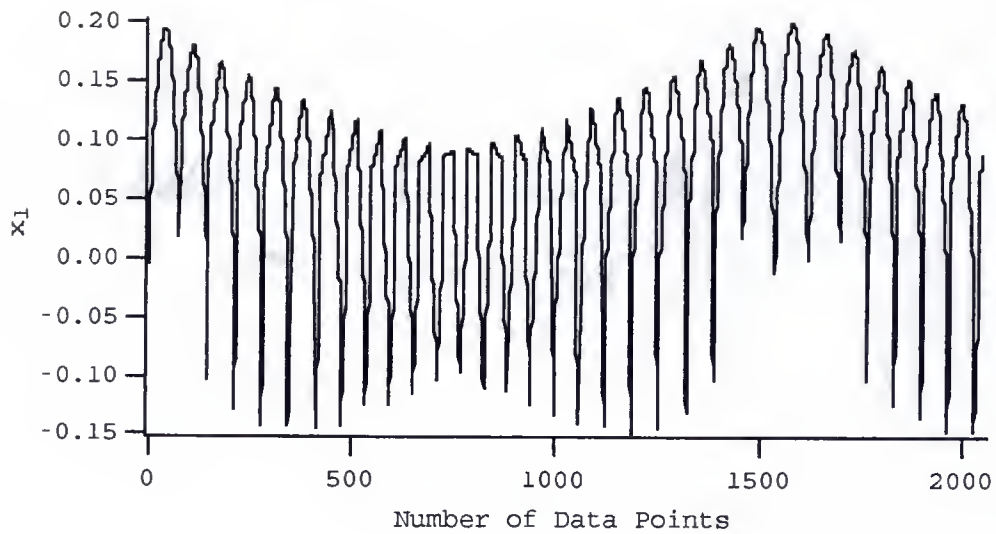
Table 3.1 Natural Frequencies (Hz) for *system a*, *system b*, and *system c*

K_2	ω_{a1}	ω_{a2}	ω_b	ω_c
570	0.9397	6.4357	5.2786	1.1457
3670	1.4076	10.9016	5.2786	2.9071

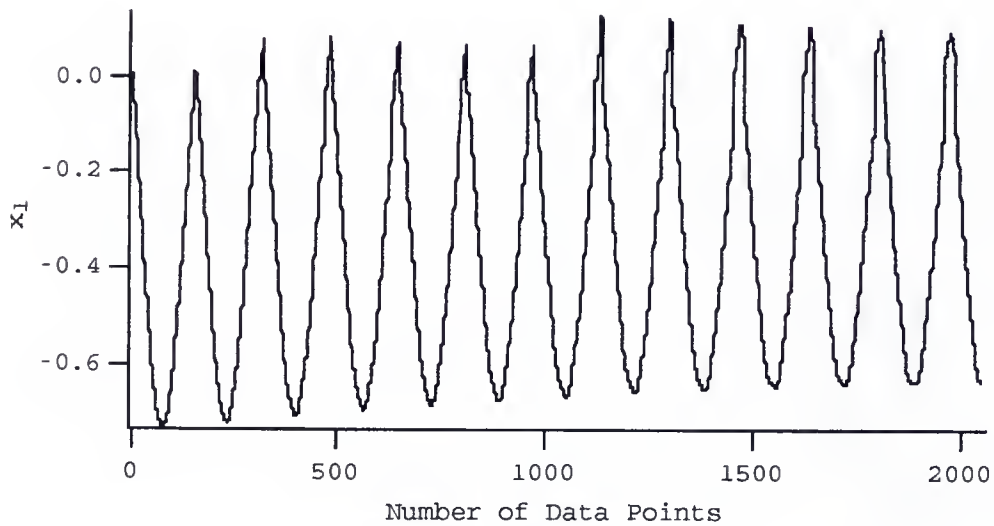
3.3.1 Case of 570N/m as Foot Stiffness (K_2)

Two Degree of Freedom System

The sets of separated data for the flight mode and the ground mode are displayed as wave forms in Figure 3.5.



(a)



(b)

Figure 3.5 Wave Forms of the Separated Data Set
 $(M_1 = 10\text{Kg}, M_2 = 1\text{Kg}, K_1 = 1000\text{N/m}, K_2 = 570\text{N/m}, \text{ and } h = 0.2\text{m})$
 (a) Flight Mode, (b) Ground Mode

The results of spectrum analyses are shown in Figure 3.6. The interesting frequencies are summarized in Table 3.2. For the flight mode, the most dominant frequency is 2.8320Hz while it is 1.1719Hz for the ground mode. Sudden changes of modes in the period diagram (Figure 3.7) can also be observed.

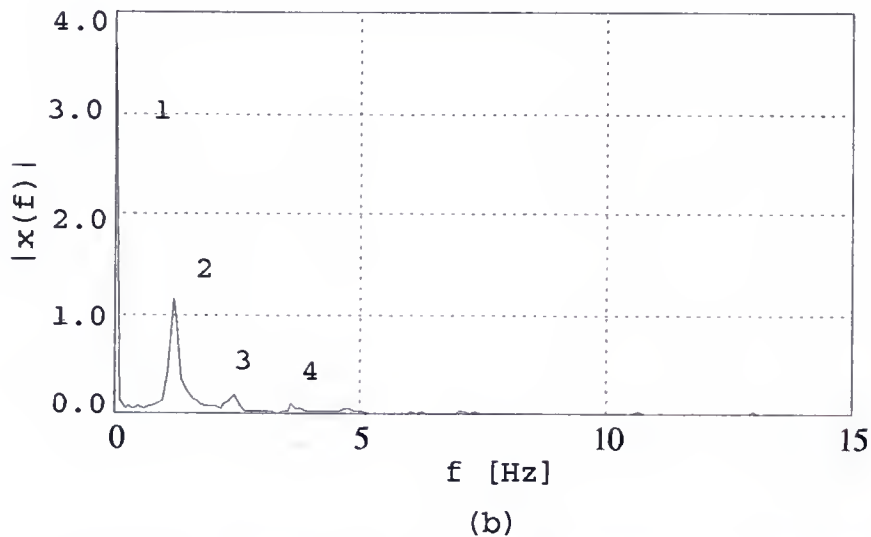
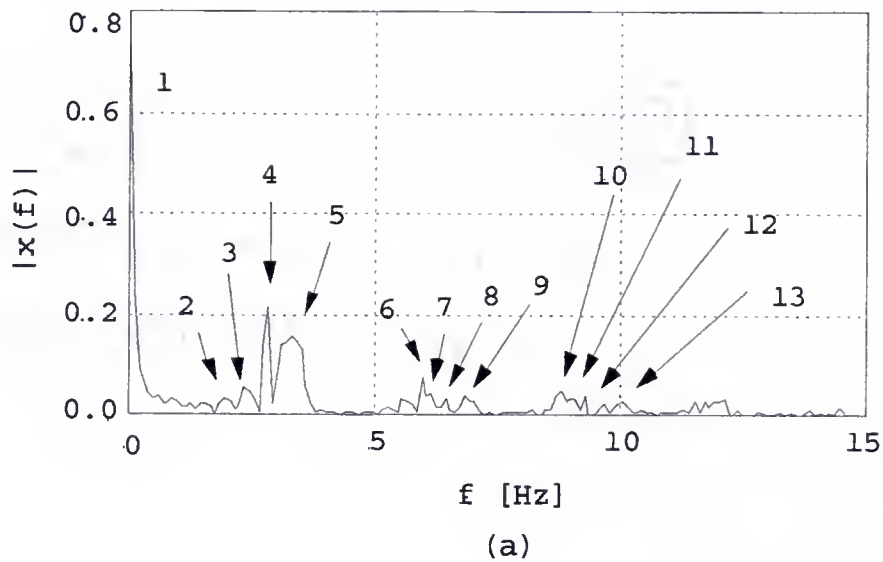


Figure 3.6 Spectrum Analysis for Two Degree of Freedom System ($M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_1 = 1000\text{N/m}$, $K_2 = 570\text{N/m}$, and $h = 0.2\text{m}$)
(a) Flight Mode, (b) Ground Mode

Table 3.2 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{dfi}	i	Ω_{dgi}
1	0	1	0
2	1.9351	2	1.1719*
3	2.3438	3	2.4414
4	2.8320*	4	3.6133
5	3.3203		
6	5.9570		
7	6.1523		
8	6.4453		
9	6.8359		
10	8.7891		
11	0.0920		
12	9.2773		
13	10.0586		

(* the most dominant frequency)

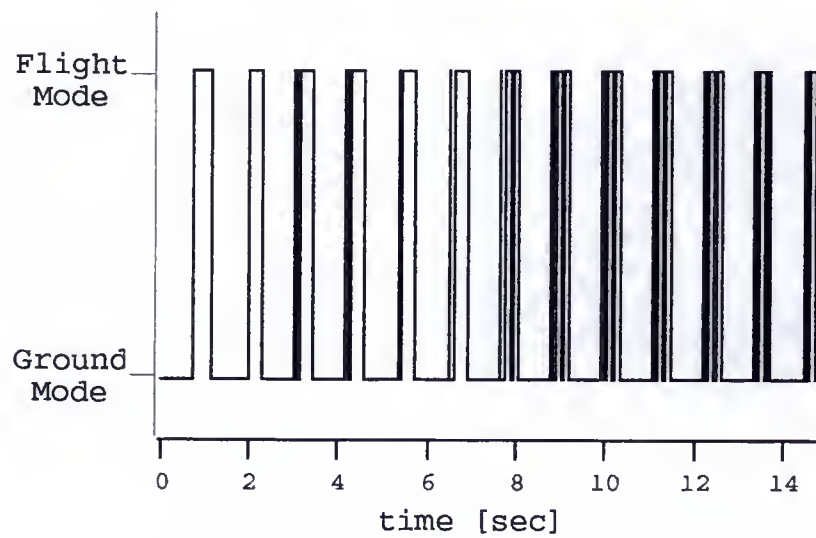


Figure 3.7 Period Diagram

One Degree of Freedom System

The total data set without separation and the separated data sets for the ground mode and the flight mode are used to obtain the dominant frequencies. First, the result of FFT for the data without separation (total data) is shown in Figure 3.8 and the dominant frequencies are listed in Table 3.3

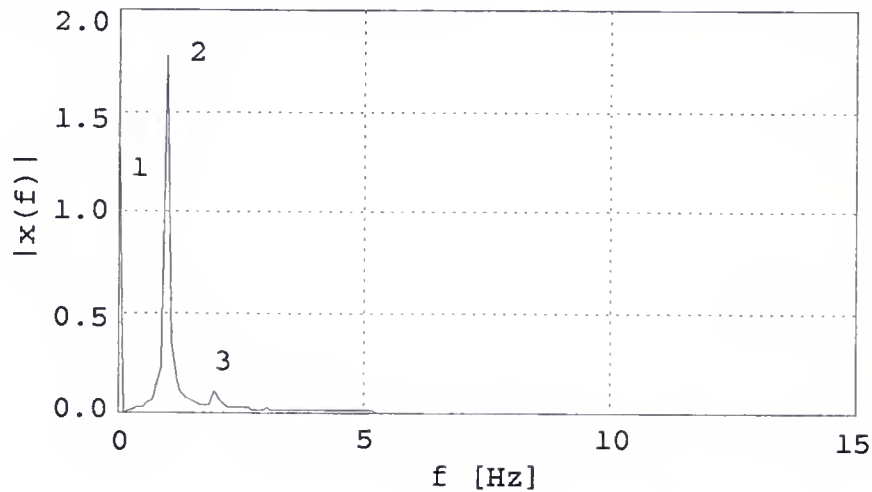


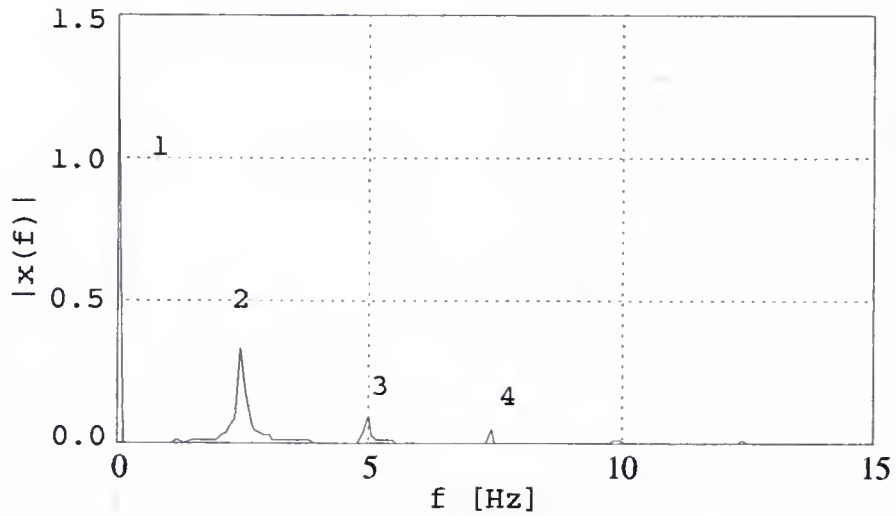
Figure 3.8 Spectrum Analysis with the Total Data

Table 3.3 Dominant Frequencies (Hz)

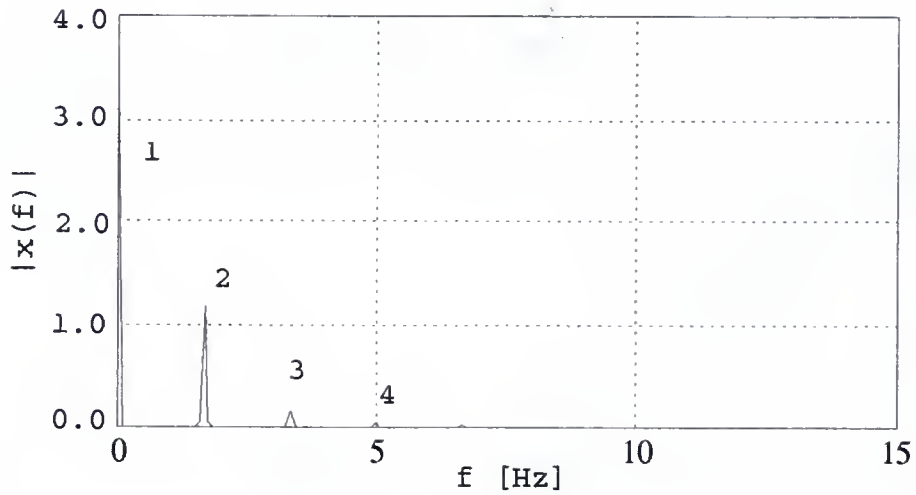
i	Ω_{sti}
1	0
2	0.9766*
3	1.9351

(* the most dominant frequency)

The most dominant frequency for the total data set for the one degree of freedom system is 0.9766Hz. The results of the spectrum analyses with the separated data sets are displayed in Figure 3.9 and Table 3.4. The most dominant



(a)



(b)

Figure 3.9 Spectrum Analysis of One Degree of Freedom System
(a) Flight Mode, (b) Ground Mode

Table 3.4 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{sfi}	i	Ω_{sgi}
1	0	1	0
2	2.4414*	2	1.6602*
3	4.9805	3	3.3203
4	7.4219	4	4.9805

(* the most dominant frequency)

frequency is 2.4414Hz for the flight mode and 1.6602Hz for the ground mode.

The frequencies of the flight mode and the ground mode can be obtained either directly from the period diagram (Figure 3.10) or analytically from the phase planes (Appendix B). The period of the ground mode is 0.603 second and that of the flight mode is 0.404 second. Thus, the corresponding frequencies of the ground mode and the flight mode become 1.6584 Hz and 2.4752 Hz, respectively (Table 3.5). Those frequencies are very close to the results of the spectrum analyses.

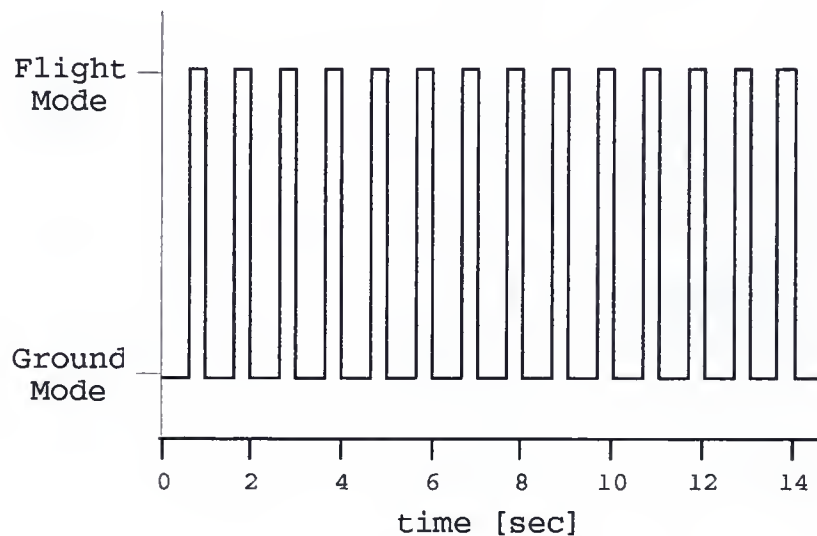


Figure 3.10 Period Diagram of One Degree of Freedom System

Table 3.5 Frequencies from Period Diagram

Mode	Frequency (Hz)
Ground	1.6584
Flight	2.4752

3.3.2 Case of 3670N/m as Foot Stiffness (K_2)

Two Degree of Freedom System

In Figure 3.11, the sets of separated data for the flight mode and the ground mode are displayed as wave forms.

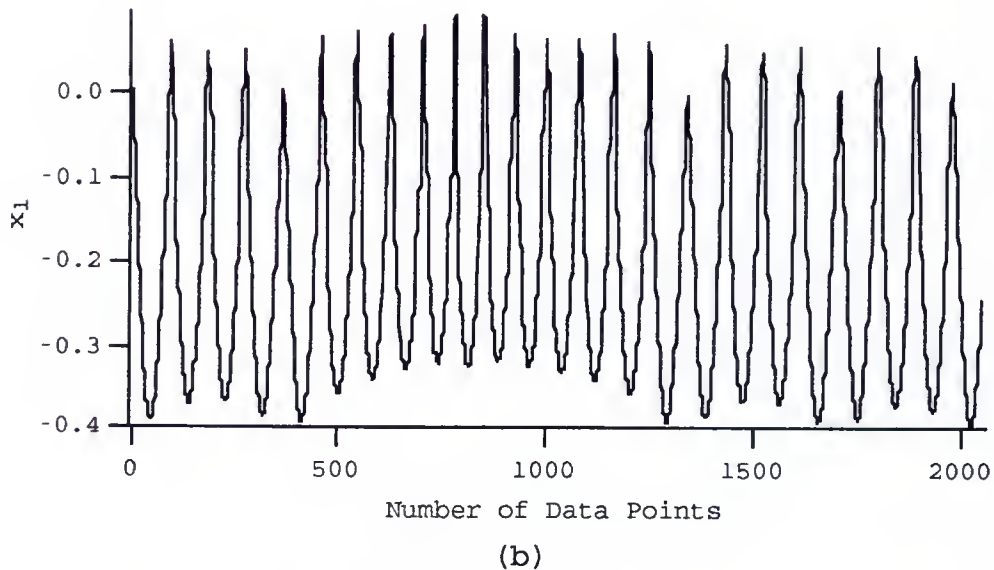
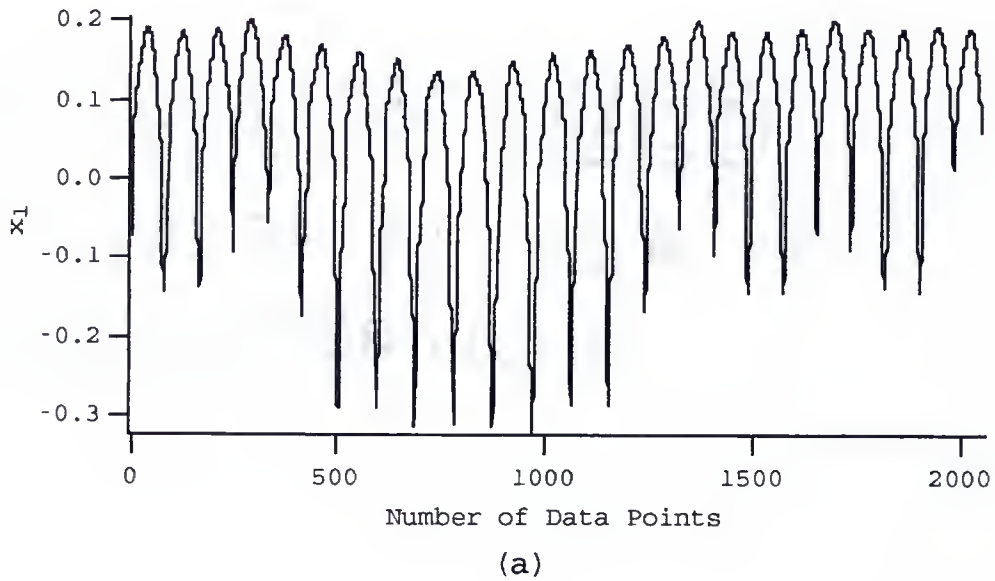


Figure 3.11 Wave Forms of the Separated Data Set
 $(M_1 = 10\text{Kg}, M_2 = 1\text{Kg}, K_1 = 1000\text{N/m}, K_2 = 3670\text{N/m}, \text{and } h = 0.2\text{m})$
 (a) Flight Mode, (b) Ground Mode

The results of spectrum analyses are displayed in Figure 3.12 and summarized in Table 3.6. It is interesting to see that the most dominant frequency is 2.2461Hz for both the flight mode and the ground mode. Abrupt changes of modes are illustrated in the period diagram (Figure 3.13).

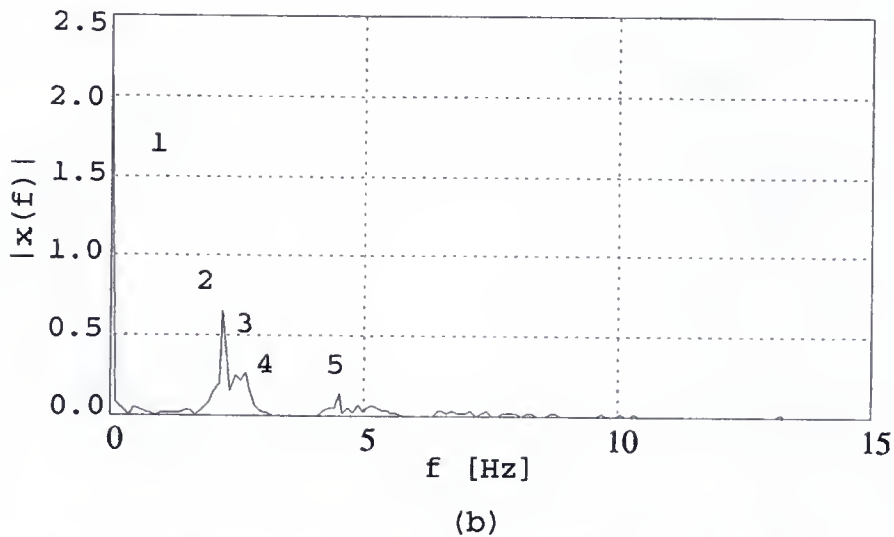
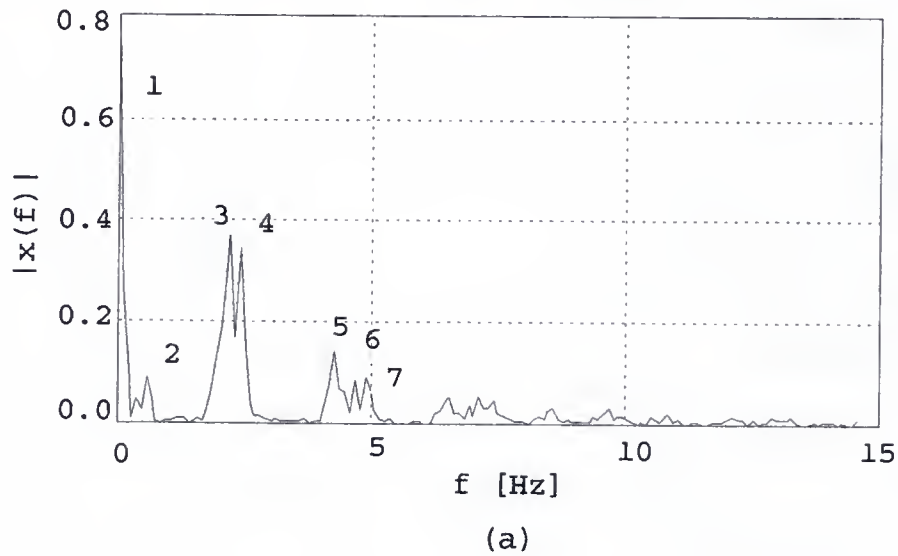


Figure 3.12 Spectrum Analysis for Two Degree of Freedom System
 $(M_1 = 10\text{Kg}, M_2 = 1\text{Kg}, K_1 = 1000\text{N/m}, K_2 = 3670\text{N/m}, \text{and } h = 0.2\text{m})$
 (a) Flight Mode, (b) Ground Mode

Table 3.6 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{dfi}	i	Ω_{dgi}
1	0	1	0
2	0.5859	2	2.2461*
3	2.2461*	3	2.4414
4	2.4414	4	2.6367
5	4.2969	5	4.4922
6	4.6875		
7	4.8828		

(* the most dominant frequency)

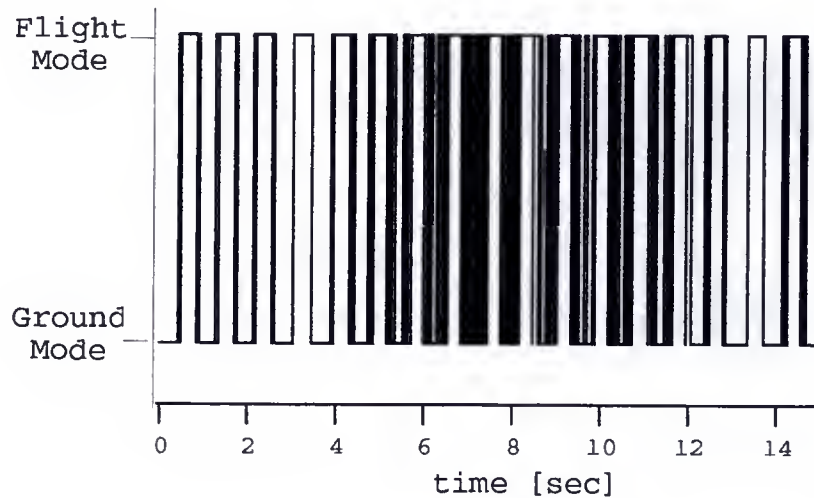


Figure 3.13 Period Diagram

One Degree of Freedom System

The data sets without and with separation for each system mode are utilized to determine the dominant frequencies by means of spectrum analysis. At first, Figure 3.14 presents the result of the spectrum analysis of the total data set. The dominant frequencies are itemized in in Table 3.7.

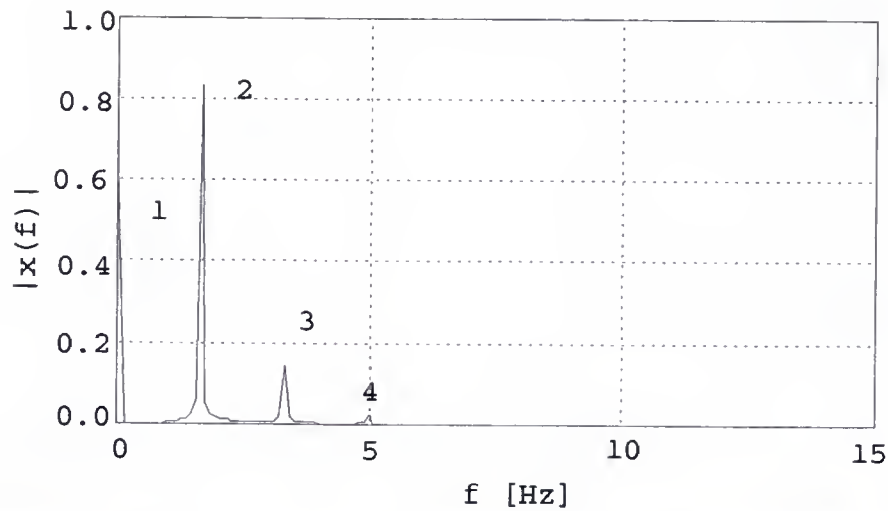


Figure 3.14 Spectrum Analysis of the Total Data Set

Table 3.7 Dominant Frequencies (Hz)

i	Ω_{sti}
1	0
2	1.6602*
3	3.3203
4	4.9805

(* the most dominant frequency)

The most dominant frequency of the total data for the one degree of freedom system is 1.6602Hz. The results of the spectrum analyses for the separated data sets are shown in Figure 3.15 and Table 3.8. The dominant frequencies are 2.4414Hz and 4.9805Hz for the flight mode and the ground mode, respectively.

The frequencies of the flight mode and the ground mode can be attained directly from the period diagram (Figure

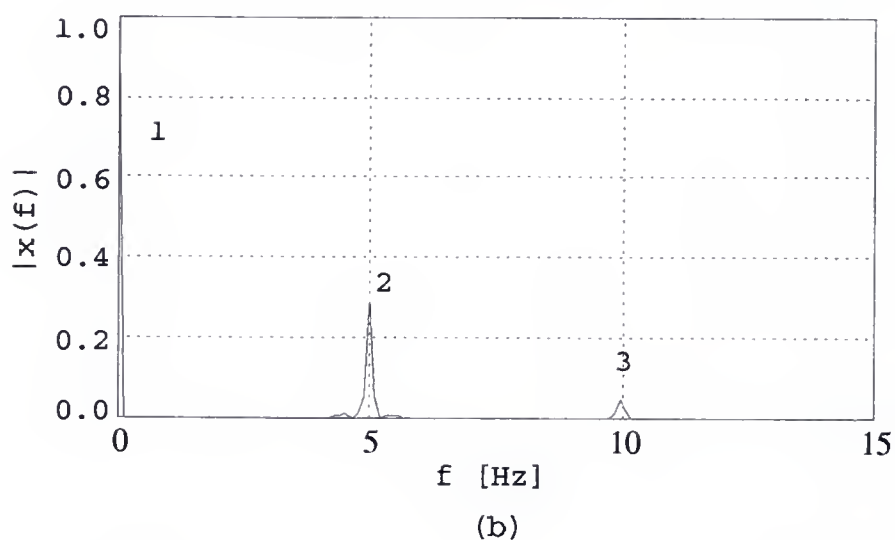
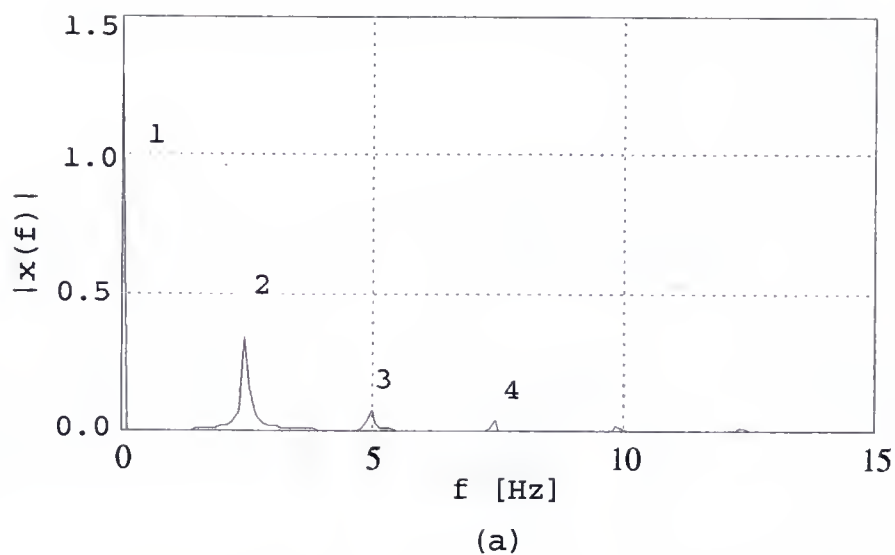


Figure 3.15 Spectrum Analysis of One Degree of Freedom System
(a) Flight Mode, (b) Ground Mode

Table 3.8 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{sfi}	i	Ω_{sgi}
1	0	1	0
2	2.4414*	2	4.9805*
3	4.9805	3	9.9609
4	7.4219		

(* the most dominant frequency)

3.16). The period of the ground mode is 0.201 second and that of the flight mode is 0.404 second. Thus, the corresponding frequencies are 4.9751Hz and 2.4752Hz for the ground mode and the flight mode, respectively (Table 3.9). These two frequencies are very close to the most dominant frequencies for each system mode obtained by the spectrum analyses.

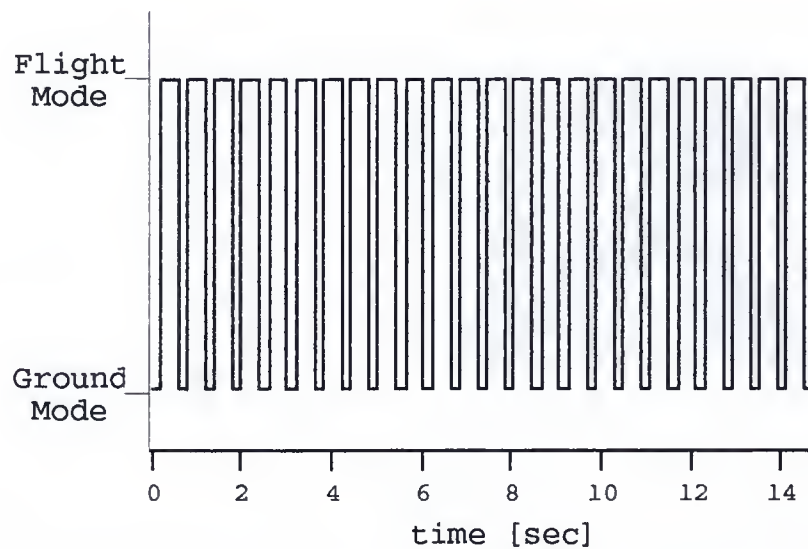


Figure 3.16 Period Diagram of One Degree of Freedom System

Table 3.9 Frequencies from the Period Diagram

Mode	Frequency (Hz)
Ground	4.9751
Flight	2.4752

3.4 Spectrum Analyses for the Chaotic Bouncing Systems with $h = 0.5\text{m}$

445N/m and 1860N/m for the foot stiffness (K_2) are

selected as system parameters for the spectrum analyses of the chaotic bouncing systems with $h = 0.5m$. According to Figure 3.3, they correspond to the high values of the sum of standard deviations and area in the phase planes. The natural frequencies with the chosen system parameters for *system a*, *system b*, and *system c* are listed in Table 3.10.

Table 3.10 Natural Frequencies (Hz) for *system a*, *system b*, and *system c*

K_2	ω_{a1}	ω_{a2}	ω_b	ω_c
445	0.8624	6.1961	5.2786	1.0123
1860	1.2755	8.5645	5.2786	2.0696

3.4.1 Case of 445N/m as Foot Stiffness (K_2)

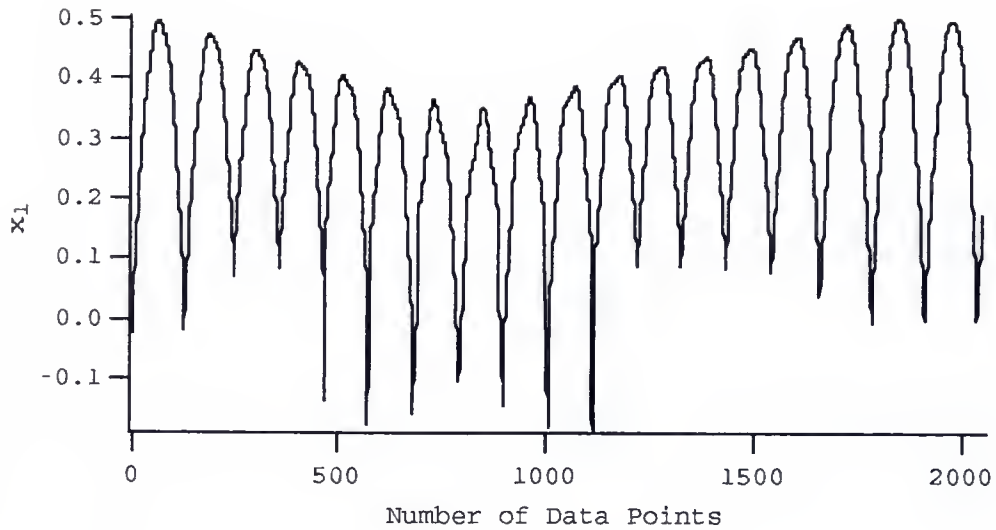
Two Degree of Freedom System

Figure 3.17 exhibits the sets of separated data for the flight mode and the ground mode as wave forms. The results of spectrum analyses are shown in Figure 3.18 and Table 3.11. For the flight mode, the most dominant frequency is 1.8555Hz while it is 1.2695Hz for the ground mode.

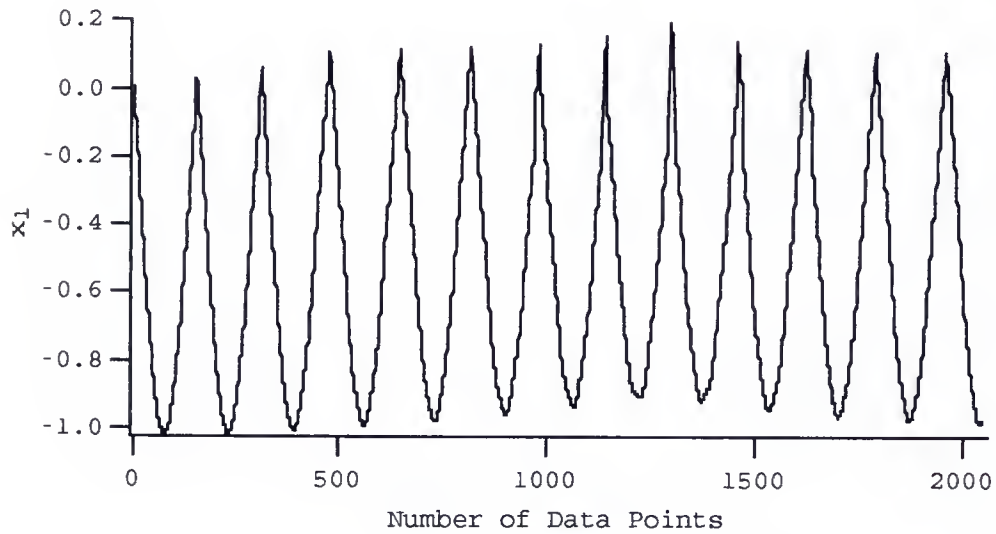
Abrupt mode changes in the period diagram (Figure 3.19), i.e., the very short intervals for each mode, can also be recognized.

One Degree of Freedom System

The complete data set without separation and the separated data sets for the ground mode and the flight mode are adopted to acquire the dominant frequencies. The result

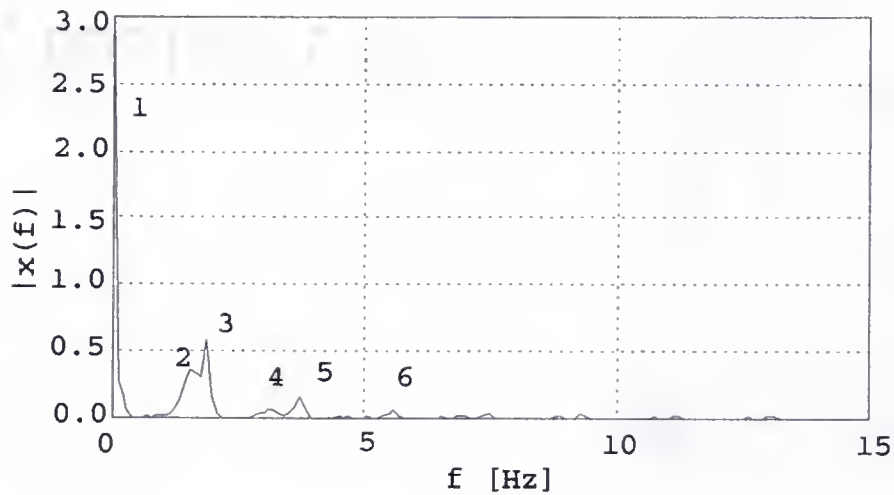


(a)

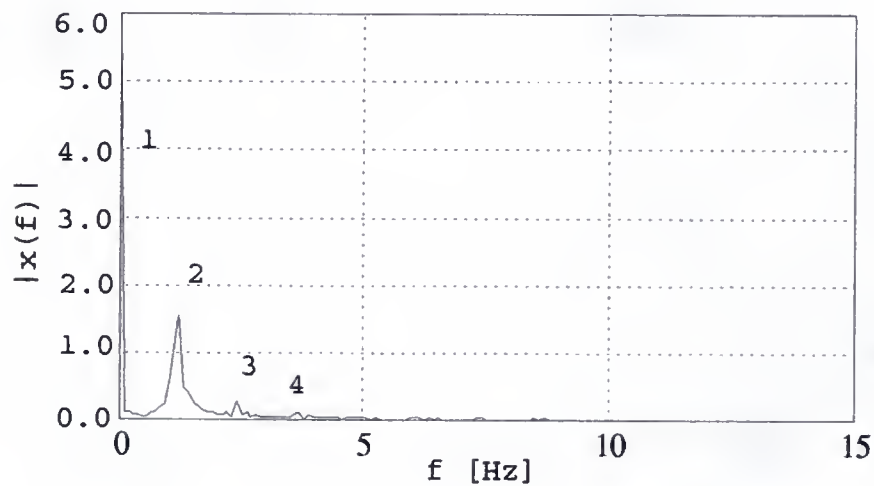


(b)

Figure 3.17 Wave Forms of the Separated Data Set
 ($M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_1 = 1000\text{N/m}$, $K_2 = 445\text{N/m}$, and $h = 0.5\text{m}$)
 (a) Flight Mode, (b) Ground Mode



(a)



(b)

Figure 3.18 Spectrum Analysis of Two Degree of Freedom System ($M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_1 = 1000\text{N/m}$, $K_2 = 445\text{N/m}$, and $h = 0.5\text{m}$)
(a) Flight Mode, (b) Ground Mode

Table 3.11 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{dfi}	i	Ω_{dgi}
1	0	1	0
2	1.5625	2	1.2695*
3	1.8555*	3	2.4414
4	3.1250	4	3.7109
5	3.7109		
6	5.5664		

(* the most dominant frequency)

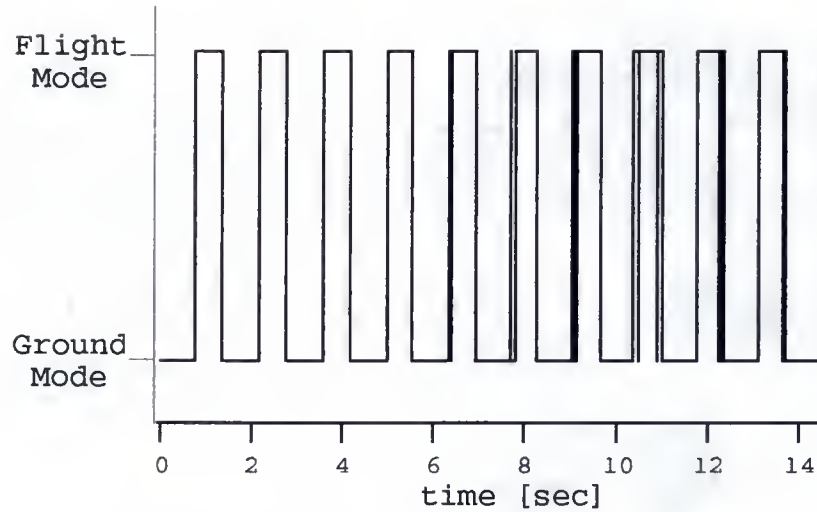


Figure 3.19 Period Diagram

of the spectrum analysis with the complete data set in in Figure 3.20 and the dominant frequencies are listed in Table 3.12.

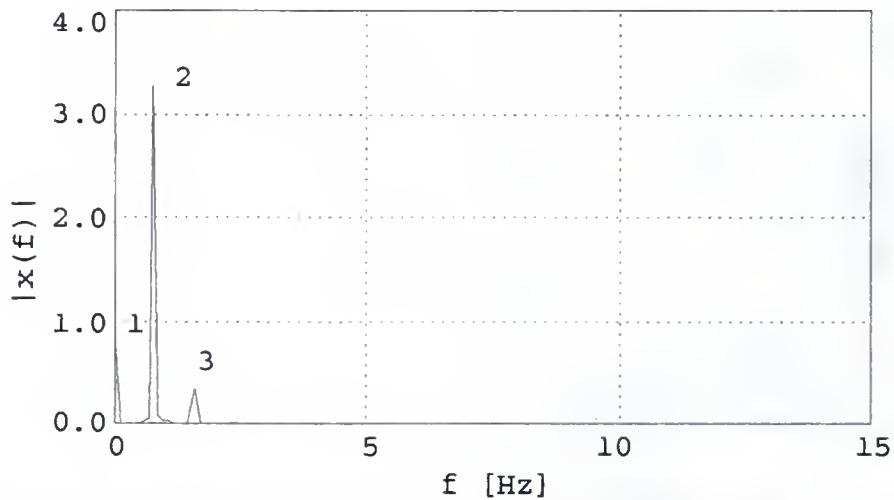


Figure 3.20 Spectrum Analysis with the Complete Data Set

Table 3.12 Dominant Frequencies (Hz)

i	Ω_{sti}
1	0
2	0.7813*
3	1.5625

(* the most dominant frequency)

The most dominant frequency with the complete data set (without separation) for the one degree of freedom system is 0.7813Hz. The results of the spectrum analyses with the separated data sets are shown in Figure 3.21. The most dominant frequency is 1.5625Hz for both the flight mode and the ground mode (Table 3.13).

Table 3.13 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{sfi}	i	Ω_{sgi}
1	0	1	0
2	1.5625*	2	1.5625*
3	3.1250	3	3.1250
4	4.6875	4	4.6875
5	6.2500		

(* the most dominant frequency)

The period of the flight mode is 0.639 second directly from the period diagram (Figure 3.22). The corresponding frequency to this period is 1.5649Hz (Table 3.14) which is approximately the same as the dominant frequency for the flight mode obtained by the spectrum analysis. It is also true for the ground mode since both periods are exactly same with selected system parameters.

3.4.2 Case of 1860N/m as Foot Stiffness (K_2)

Two Degree of Freedom System

The separated data sets for the flight mode and the ground mode are shown as wave forms in Figure 3.23. The

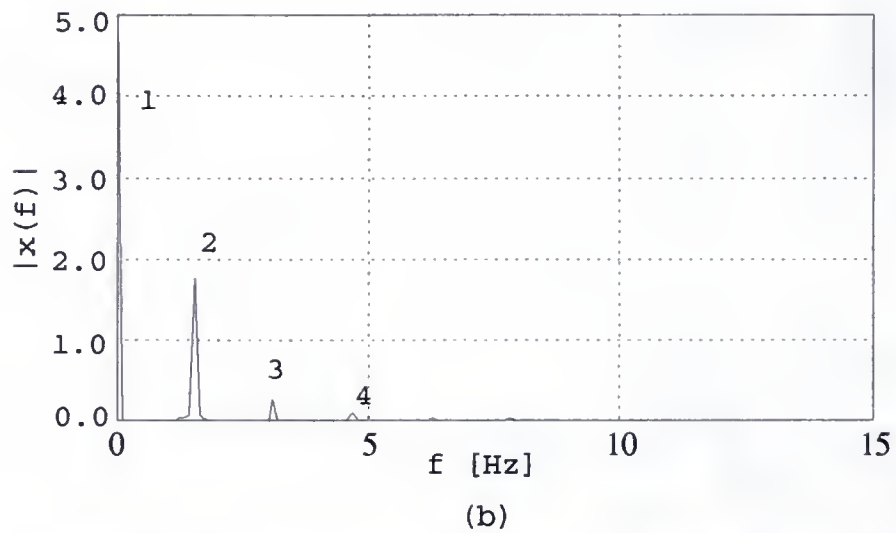
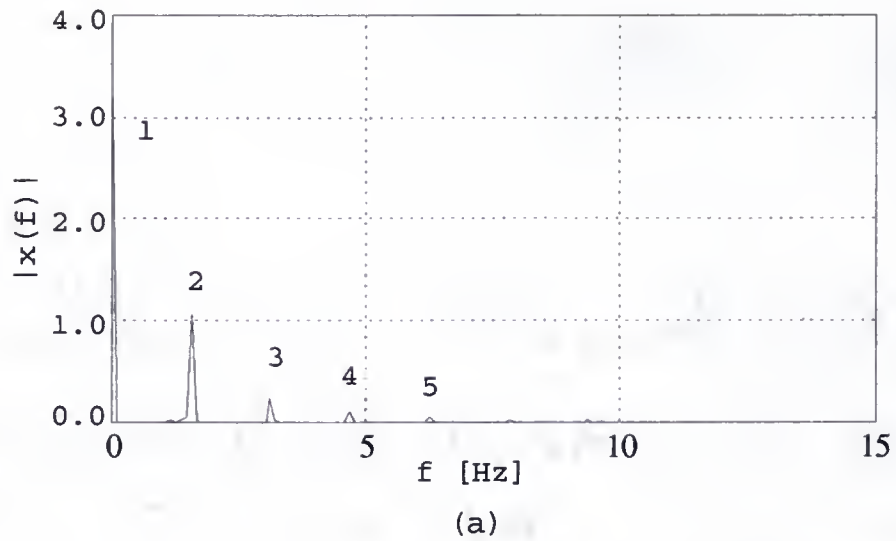


Figure 3.21 Spectrum Analyses of One Degree of Freedom System
(a) Flight Mode, (b) Ground Mode)

Table 3.14 Frequencies from Period Diagram

Mode	Frequency (Hz)
Ground	1.5649
Flight	1.5649

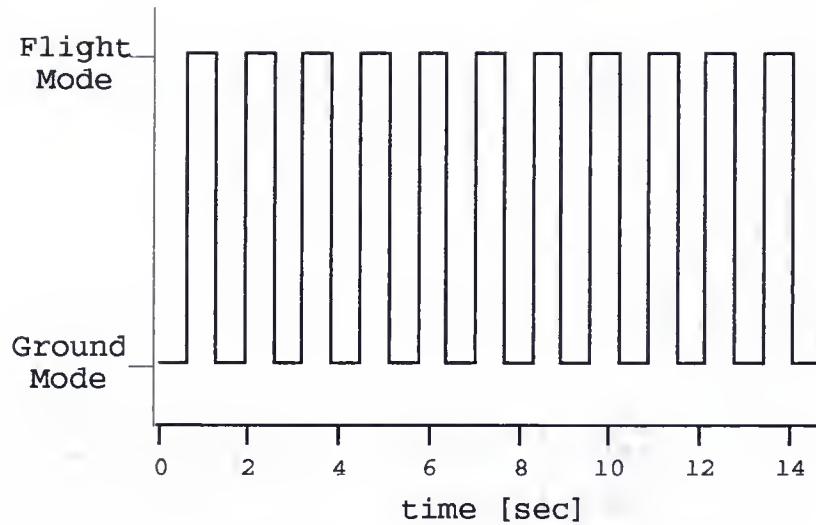


Figure 3.22 Period Diagram

results of the spectrum analyses with these separated data sets are displayed in Figure 3.24. For the flight mode, the most dominant frequency is 1.5625Hz while it is 2.2461Hz for the ground mode according to Table 3.15. Sudden changes of system modes in the period diagram (Figure 3.25) can also be observed.

One Degree of Freedom System

The data sets with and without separation for the flight mode and the ground mode are used to find the dominant frequencies by the spectrum analysis. The results of the spectrum analysis for the data set without separation are shown in Figure 3.26 and Table 3.16. The most dominant frequency in this case is 1.0742Hz. The spectrum analysis results for the data sets with separation for each system mode are shown in Figure 3.27 and Table 3.17.

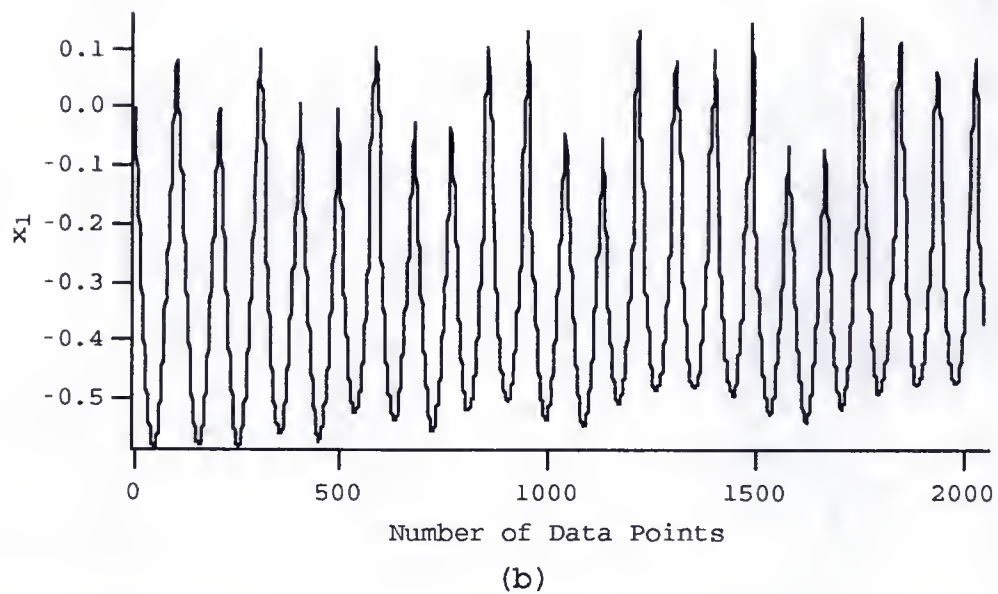
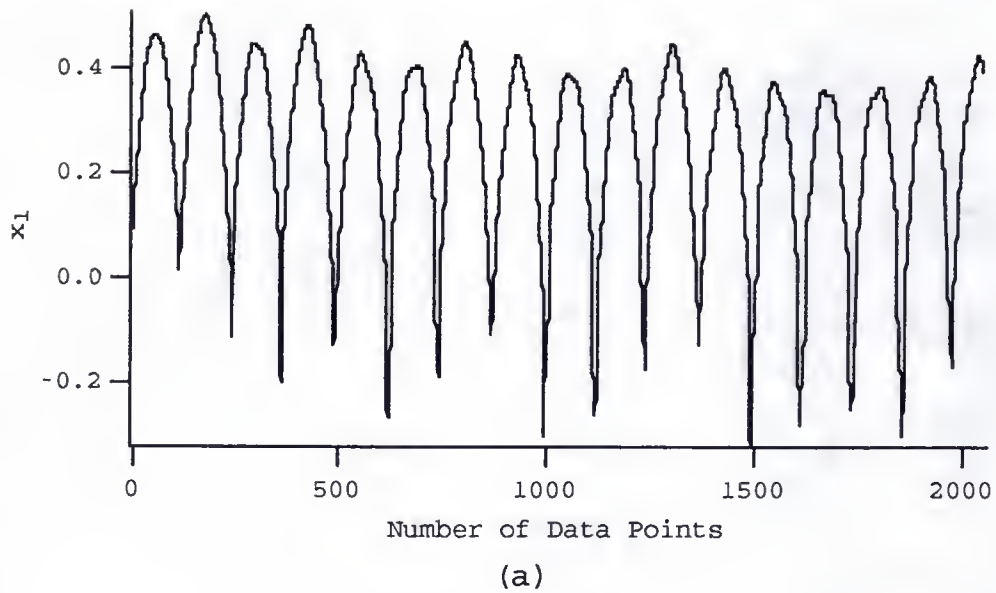


Figure 3.23 Wave forms of the Separated Data Set
 $(M_1 = 10\text{Kg}, M_2 = 1\text{Kg}, K_1 = 1000\text{N/m}, K_2 = 1860\text{N/m}, \text{and } h = 0.5\text{m})$
 (a) Flight Mode, (b) Ground Mode

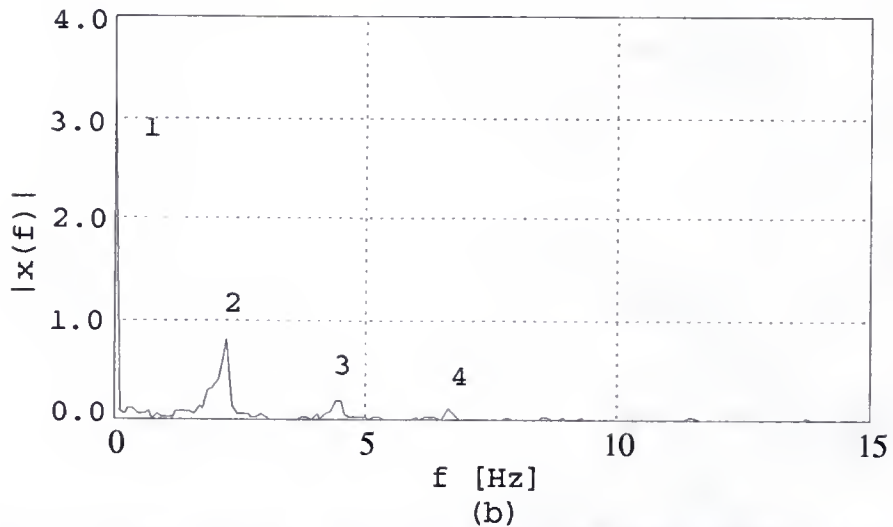
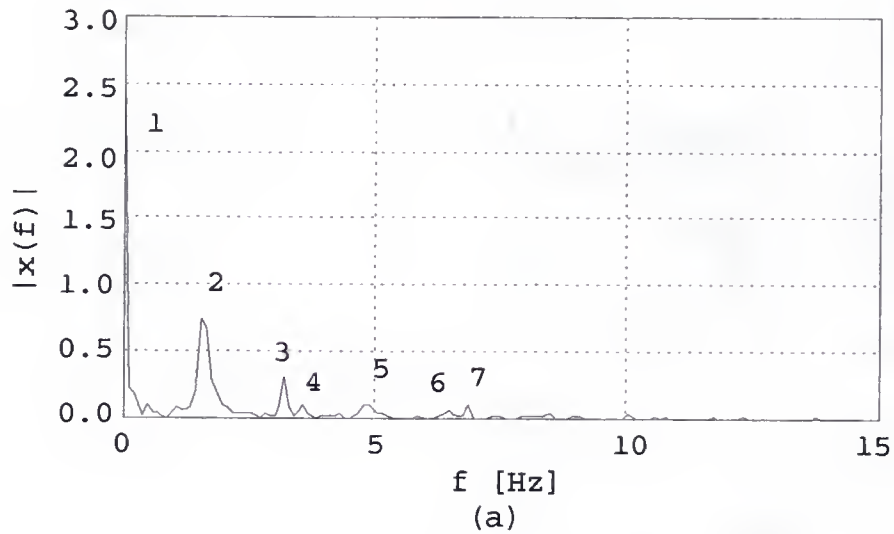


Figure 3.24 Spectrum Analyses of Two Degree of Freedom System ($M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_1 = 1000\text{N/m}$, $K_2 = 1860\text{N/m}$, and $h = 0.5\text{m}$)
(a) Flight Mode, (b) Ground Mode

Table 3.15 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{dfi}	i	Ω_{dgi}
1	0	1	0
2	1.5625*	2	2.2461*
3	3.2227	3	4.4922
4	3.6133	4	6.6406
5	4.7852		
6	6.4453		
7	6.8359		

(* the most dominant frequency)

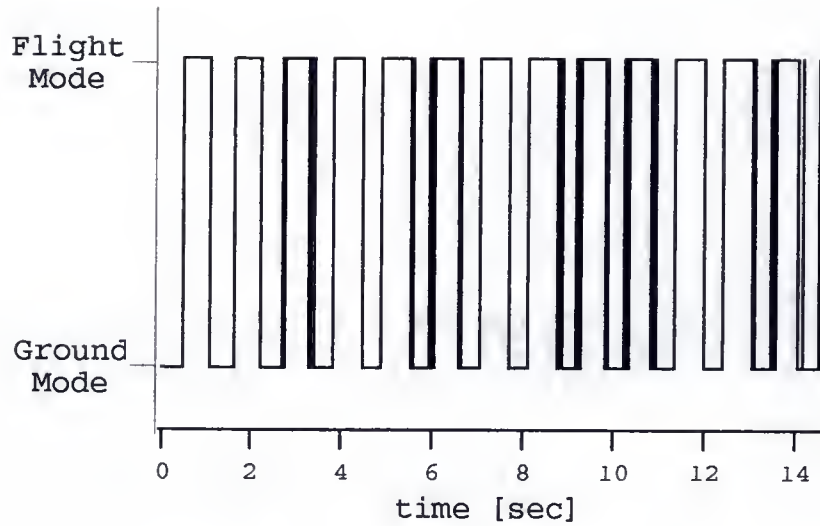


Figure 3.25 Period Diagram

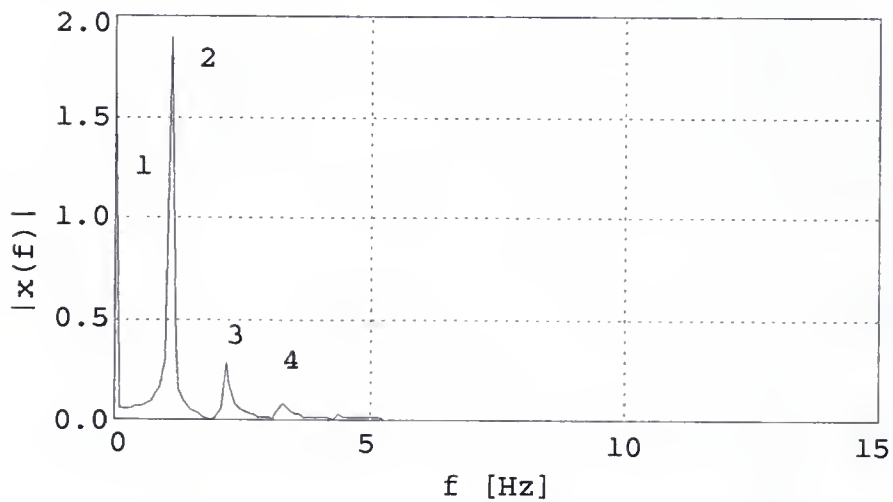


Figure 3.26 Spectrum Analysis of the Data Set without Separation

Table 3.16 Dominant Frequencies (Hz)

i	Ω_{sti}
1	0
2	1.0742*
3	2.1484
4	3.3203

(* the most dominant frequency)

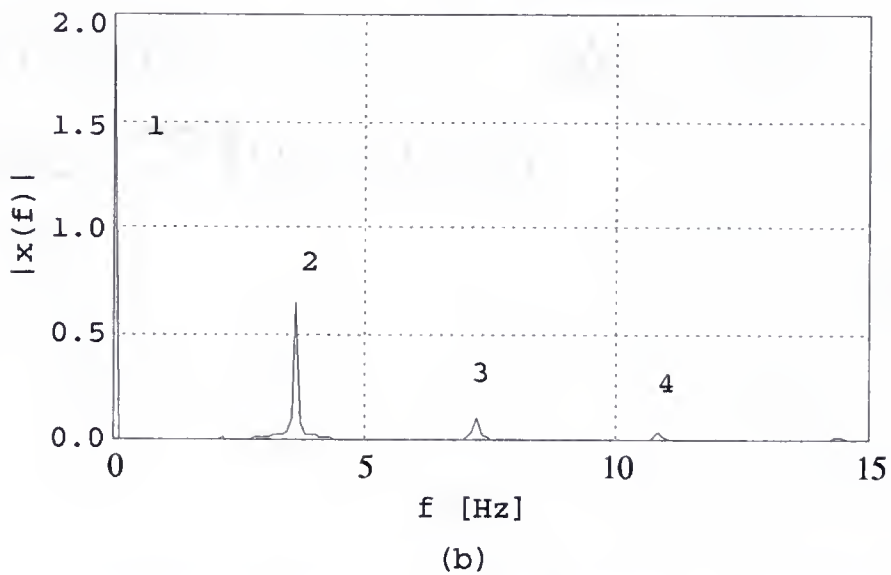
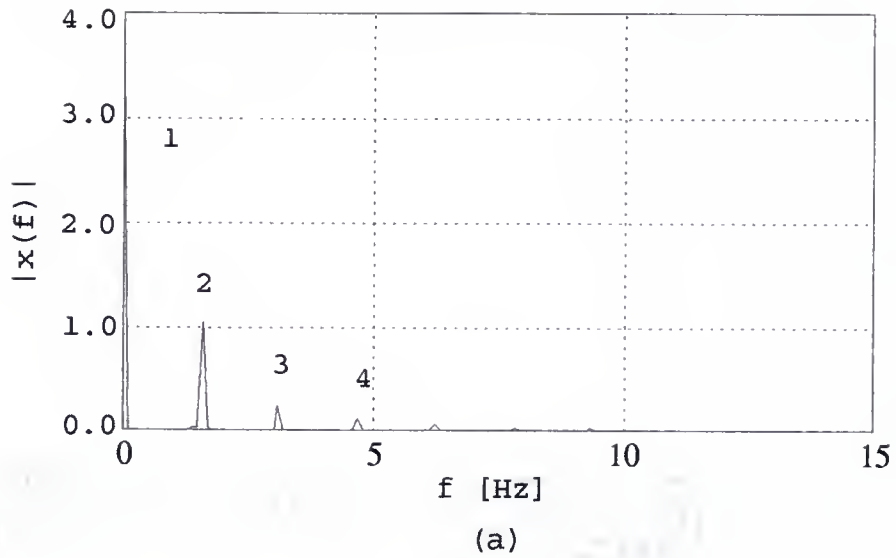


Figure 3.27 Spectrum Analysis of One Degree of Freedom System
(a) Flight Mode, (b) Ground Mode

Table 3.17 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{sfi}	i	Ω_{sgi}
1	0	1	0
2	1.5625*	2	3.6133*
3	3.1250	3	7.2266
4	4.6875	4	10.8398

(* the most dominant frequency)

The dominant frequencies are 1.5625Hz and 3.6133Hz for the flight mode and the ground mode, respectively (Table 3.17).

The period of the ground mode is 0.277 second and that of the flight mode is 0.639 second as taken directly from the period diagram (Figure 3.28). Therefore, the corresponding frequency for the ground mode is 3.6101Hz while it is 1.5625Hz for the flight mode (Table 3.18).

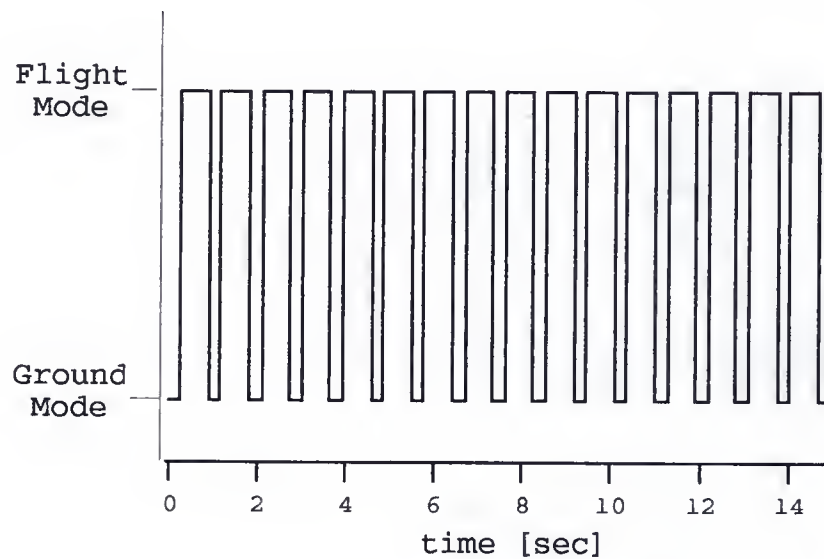


Figure 3.28 Period Diagram

Table 3.18 Frequencies from the Period Diagram

Mode	Frequency (Hz)
Ground	3.6101
Flight	1.5625

3.5 Spectrum Analyses for the Chaotic Bouncing

Systems with $h = 1.0\text{m}$

2010N/m and 2745N/m for the foot stiffness (K_2) are selected as system parameters for the spectrum analyses of the chaotic bouncing systems with $h = 1.0\text{m}$. According to Figure 3.4, they correspond the high values of the sum of standard deviations and area in the phase planes. The natural frequencies with the chosen system parameters for *system a*, *system b*, and *system c* are listed in Table 3.19.

Table 3.19 Natural Frequencies (Hz) for *system a*, *system b*, and *system c*

K_2	ω_{a1}	ω_{a2}	ω_b	ω_c
2010	1.2933	8.7809	5.2786	2.1514
2745	1.3577	9.7751	5.2786	2.5142

3.5.1 Case of 2010N/m as Foot Stiffness (K_2)

Two Degree of Freedom System

Figure 3.29 exhibits the separated data sets for the flight mode and the ground mode as wave forms.

With these data sets, the spectrum analyses are carried on to obtain the dominant frequencies. The results are shown in Figure 3.30 and they are summarized in Table 3.20. The most dominant frequency for the flight mode is 1.1719Hz while it is 2.5391Hz for the ground mode. Rapid changes between the ground mode and the flight mode exist in the period diagram (Figure 3.31).

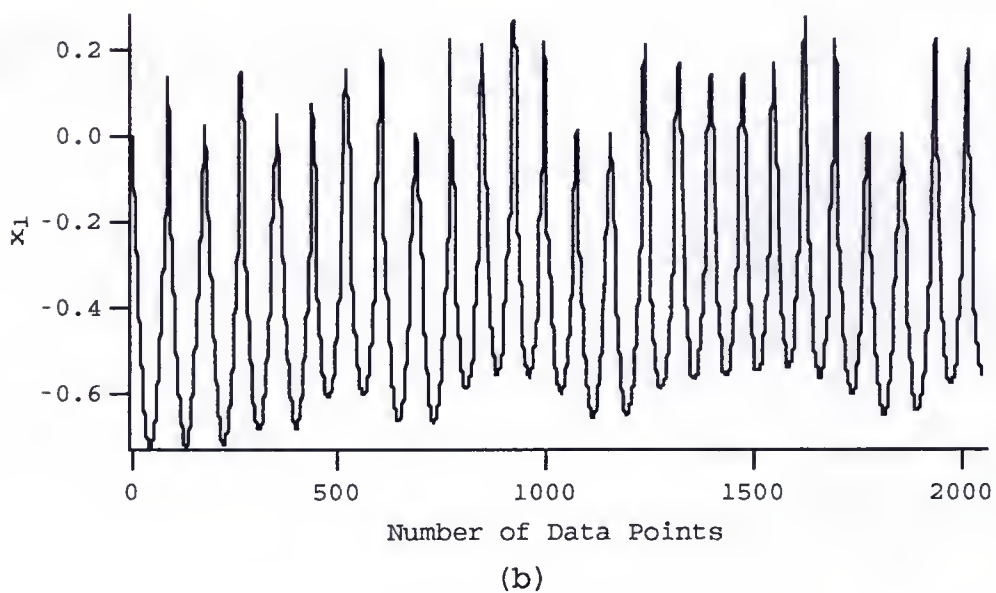
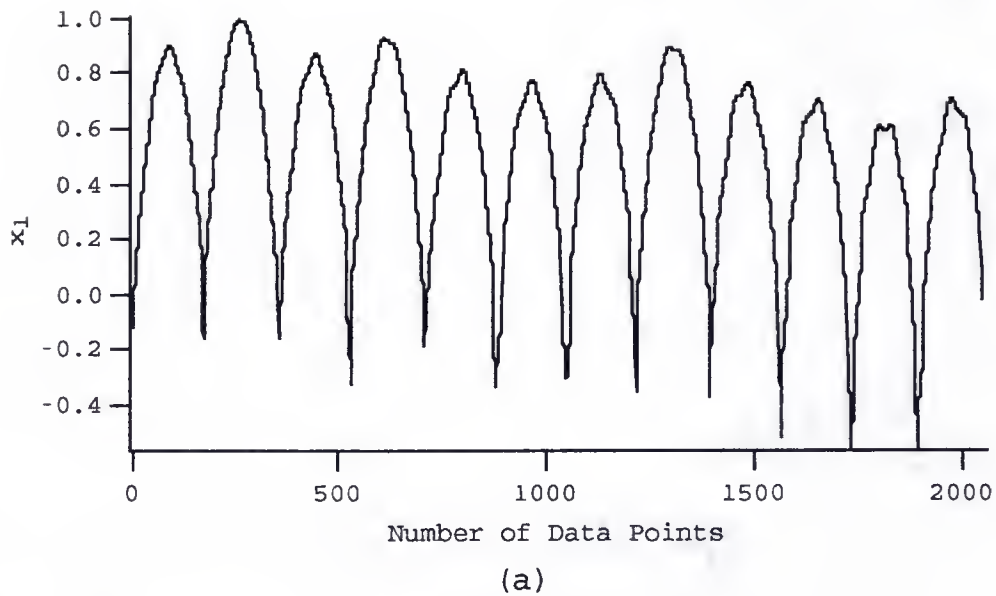


Figure 3.29 Wave Forms of the Separated Data Set
 $(M_1 = 10\text{Kg}, M_2 = 1\text{Kg}, K_1 = 1000\text{N/m}, K_2 = 2010\text{N/m}, \text{and } h = 1.0\text{m})$
 (a) Flight Mode, (b) Ground Mode

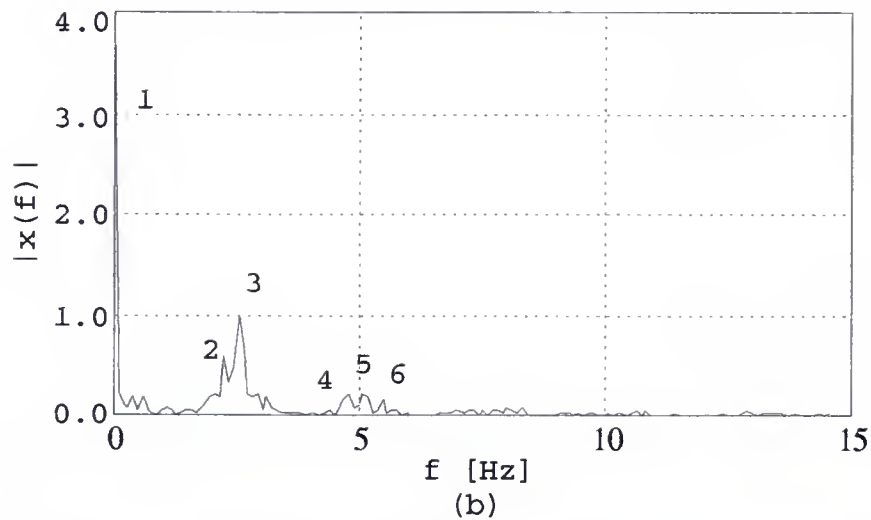
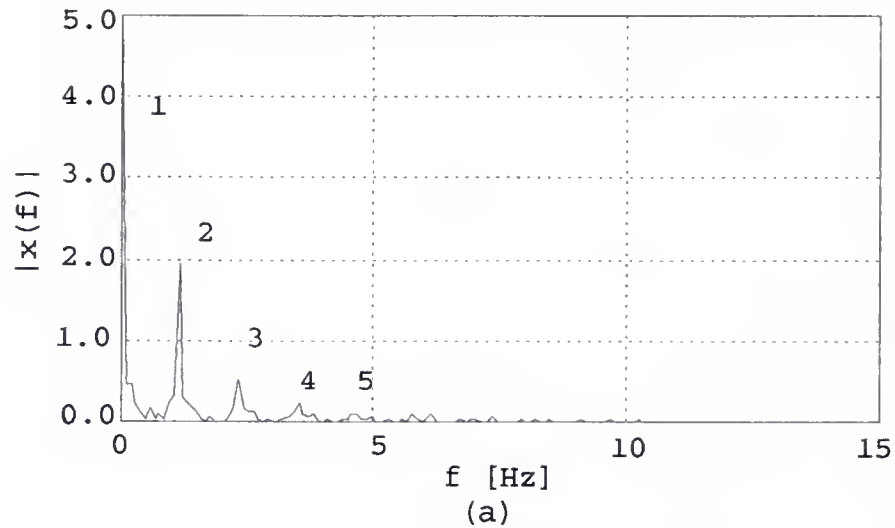


Figure 3.30 Spectrum Analysis of Two Degree of Freedom System ($M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_1 = 1000\text{N/m}$, $K_2 = 2010\text{N/m}$, and $h = 1.0\text{m}$)
(a) Flight Mode, (b) Ground Mode

Table 3.20 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{dfi}	i	Ω_{dgi}
1	0	1	0
2	1.1719*	2	2.2461
3	2.3438	3	2.5391*
4	3.5156	4	4.7852
		5	5.0781
		6	5.4688

(* the most dominant frequency)

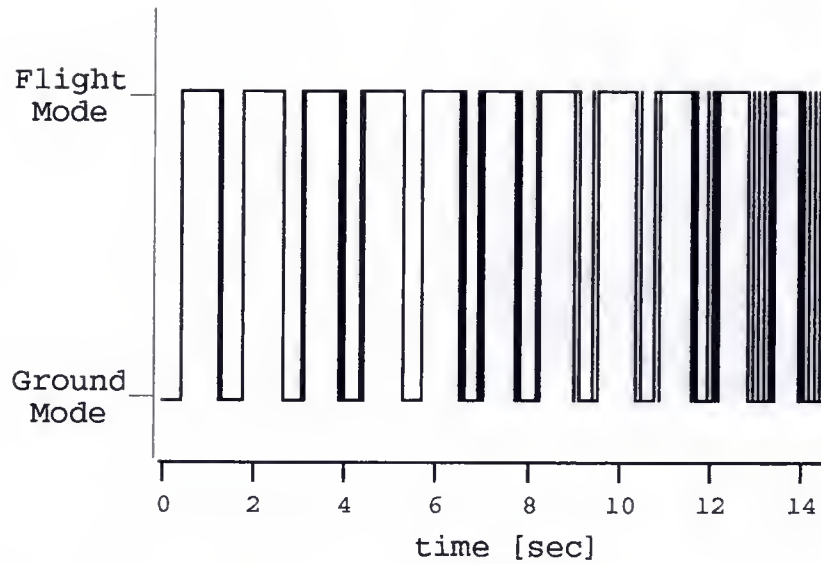


Figure 3.31 Period Diagram

One Degree of Freedom System

The data set without separation and the data sets with separation for the flight mode and the ground mode are employed to get the dominant frequencies by the spectrum analysis. The result of spectrum analysis for the data set without separation is displayed in Figure 3.32.

The most dominant frequency of the data set without separation for the one degree of freedom system is 0.8789Hz (Table 3.21).

Table 3.21 Dominant Frequencies (Hz)

i	Ω_{sti}
1	0
2	0.8789*
3	1.7578
4	2.5391
5	3.4180

(* the most dominant frequency)

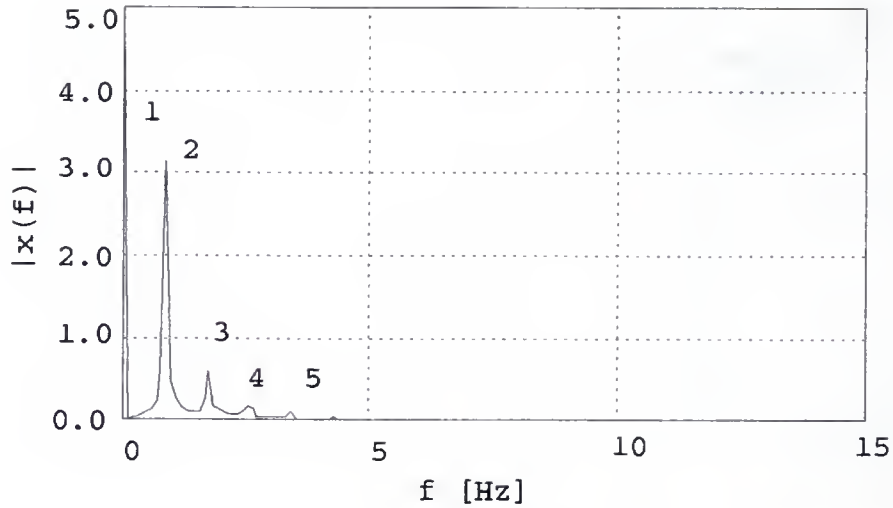


Figure 3.32 Spectrum Analysis of the Data Set Without Separation

The results of the spectrum analyses for the data sets with separation are shown in Figure 3.33. The most dominant frequencies are 1.0742Hz and 3.9063Hz for the flight mode and the ground mode, respectively (Table 3.22).

The period of the ground mode is 0.256 second and that of the flight mode is 0.903 second as taken directly from the period diagram (Figure 3.34). Therefore, the corresponding frequency for the ground mode is 3.9063Hz while it is 1.1074Hz for the flight mode (Table 3.23).

Table 3.22 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{sfi}	i	Ω_{sgi}
1	0	1	0
2	1.0742*	2	3.9063*
3	2.2461	3	7.8125
4	3.3203	4	11.7188

(* the most dominant frequency)

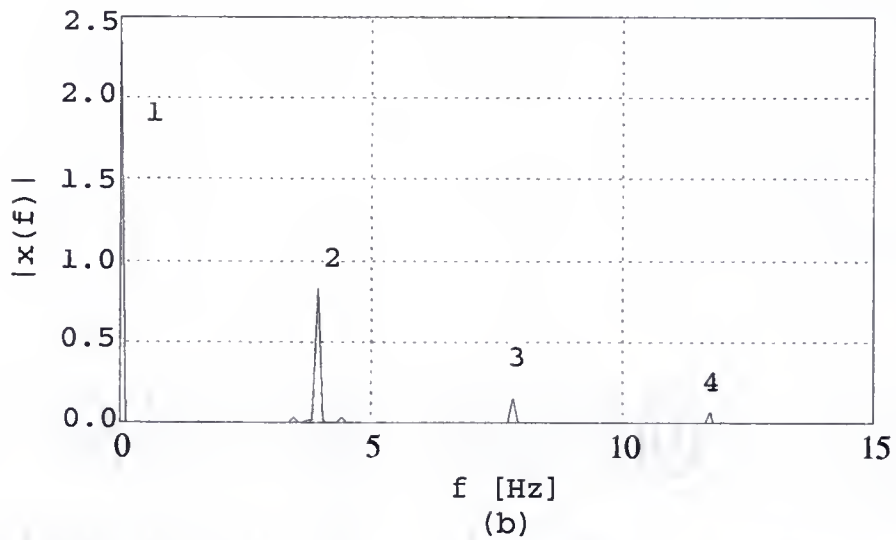
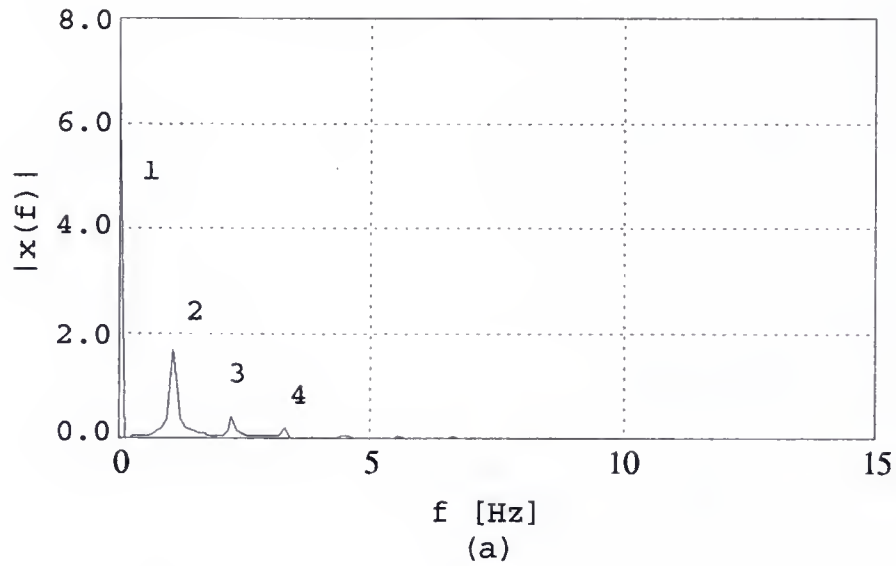


Figure 3.33 Spectrum Analysis of One Degree of Freedom System
(a) Flight Mode, (b) Ground Mode

Table 3.23 Frequencies From Period Diagram

Mode	Frequency (Hz)
Ground	3.9063
Flight	1.1074

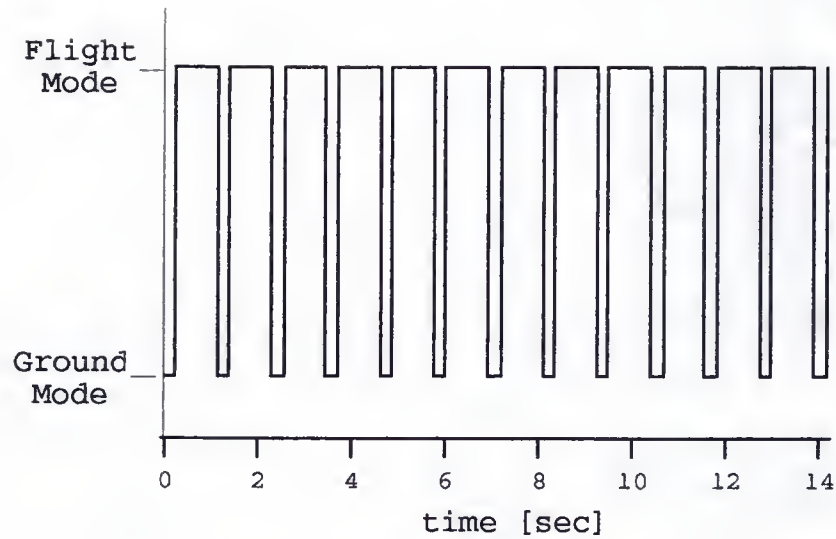


Figure 3.34 Period Diagram

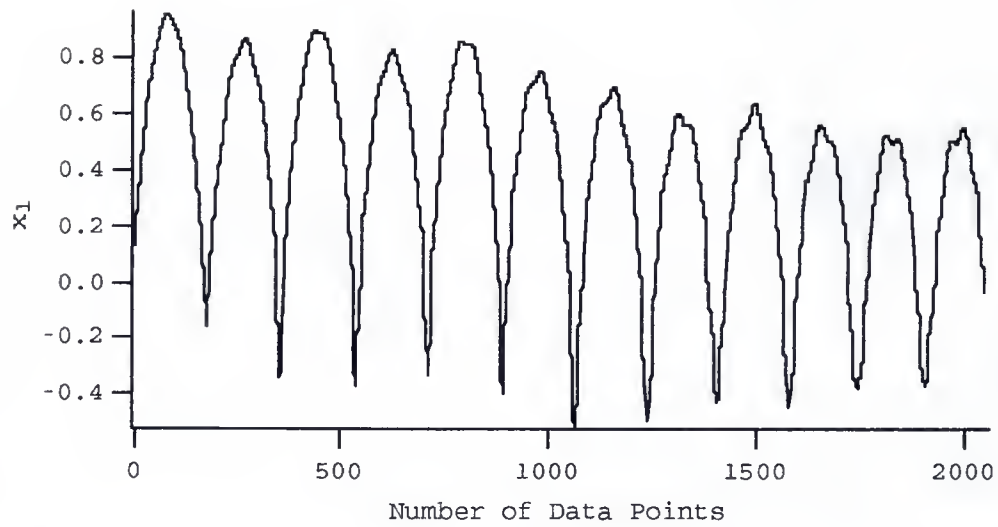
3.5.2 Case of 2745N/m as Foot Stiffness (K_2)

Two Degree of Freedom System

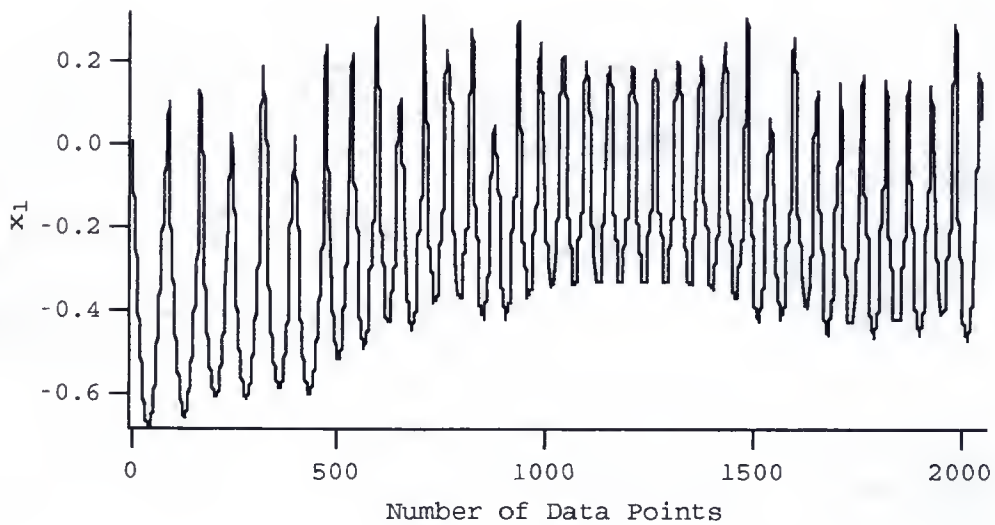
The separated data sets for the flight mode and the ground mode are displayed in Figure 3.35 as wave forms.

The results of the spectrum analyses with these data sets are displayed in Figure 3.36. The most dominant frequency for the flight mode is 1.1719Hz while it is 3.6133Hz for the ground mode (Table 3.24).

There are sudden changes between the flight mode and the ground mode in the period diagram (Figure 3.37).



(a)



(b)

Figure 3.35 Wave Forms of the Separated Data Set
 $(M_1 = 10\text{Kg}, M_2 = 1\text{Kg}, K_1 = 1000\text{N/m}, K_2 = 2745\text{N/m}, \text{and } h = 1.0\text{m})$
 (a) Flight Mode, (b) Ground Mode

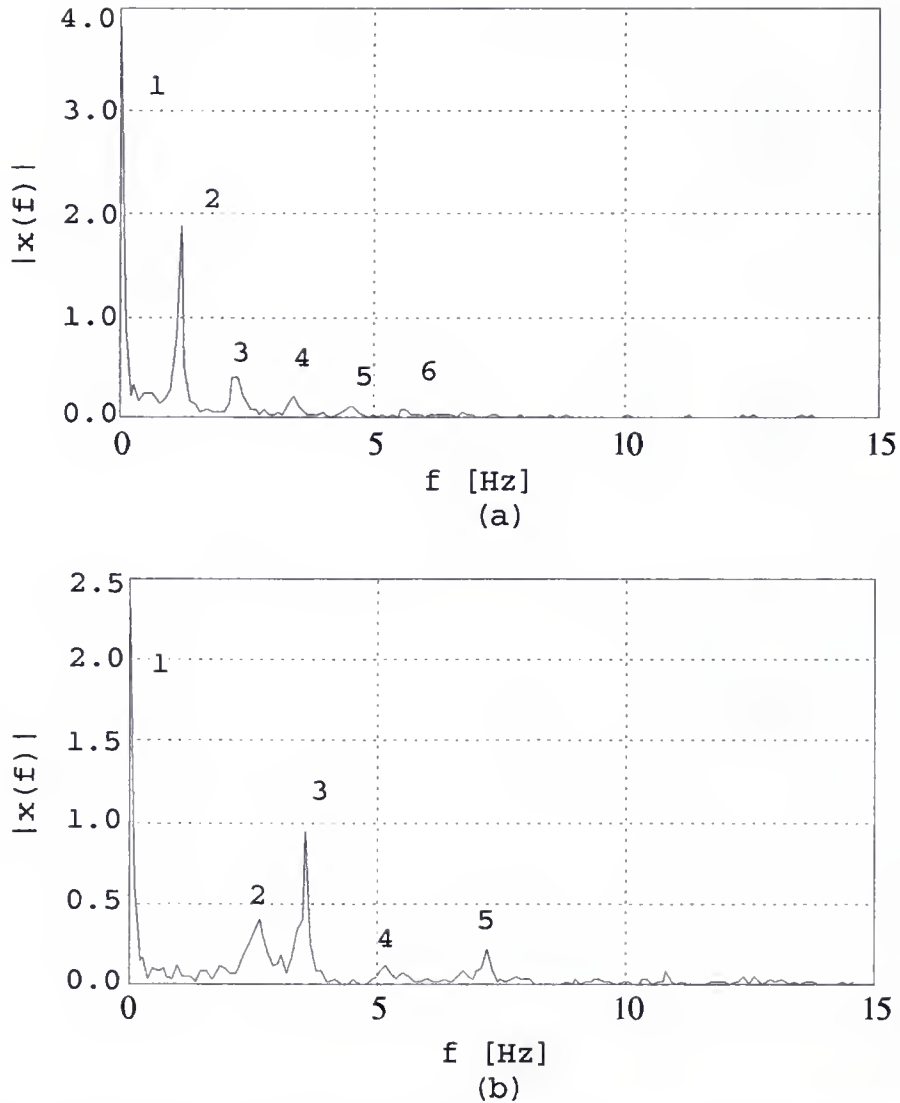


Figure 3.36 Spectrum Analysis of Two Degree of Freedom System ($M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_1 = 1000\text{N/m}$, $K_2 = 2745\text{N/m}$, and $h = 1.0\text{m}$)
(a) Flight Mode, (b) Ground Mode

Table 3.24 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{dfi}	i	Ω_{dgi}
1	0	1	0
2	1.1719*	2	2.6397
3	2.2461	3	3.6133*
4	3.4180	4	5.1758
5	4.4922	5	7.1289
6	5.6641		

(* the most dominant frequency)



Figure 3.37 Period Diagram

One Degree of Freedom System

In order to get the dominant frequencies for the selected one degree of freedom system, spectrum analyses are carried out for the data set without separation and the data set with the separation. Figure 3.38 displays the result of the spectrum analysis of the data set without separation. The most dominant frequency in this case is 0.8789Hz (Table 3.25).

Table 3.25 Dominant Frequencies

i	Ω_{sti}
1	0
2	0.8789*
3	1.7578
4	2.7344
5	3.6133

(* the most dominant frequency)

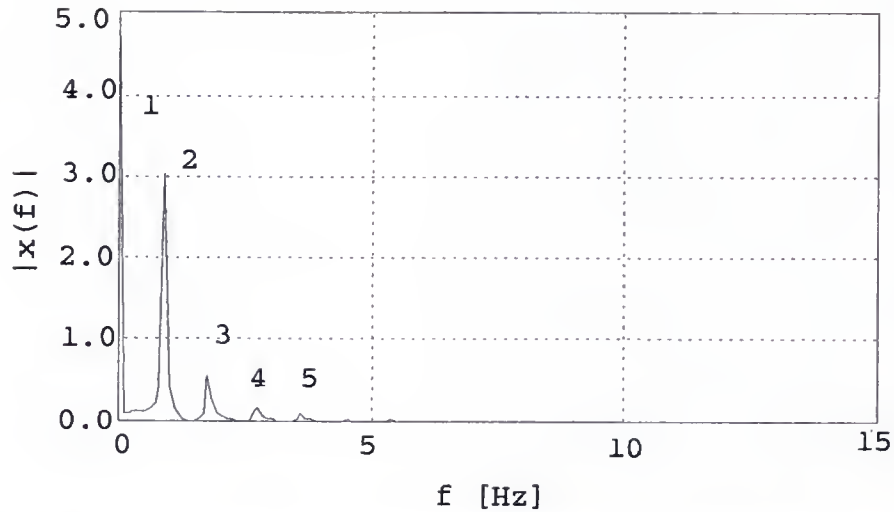


Figure 3.38 Spectrum Analysis of the Data Set Without Separation

The results of the spectrum analyses for the data sets with separation are shown in Figure 3.39. The most dominant frequencies are 1.0742Hz and 4.5898Hz for the flight mode and the ground mode, respectively (Table 3.26).

The period of the ground mode is 0.216 second and that of the flight mode is 0.903 second as taken directly from the period diagram (Figure 3.40). Thus, the corresponding frequency for the ground mode is 4.6296Hz while it is 1.1074Hz for the flight mode (Table 3.27).

Table 3.26 Dominant Frequencies (Hz)

Flight Mode		Ground Mode	
i	Ω_{sfi}	i	Ω_{sg_i}
1	0	1	0
2	1.0742*	2	4.5898*
3	2.2461	3	9.2773
4	3.3203		

(* the most dominant frequency)

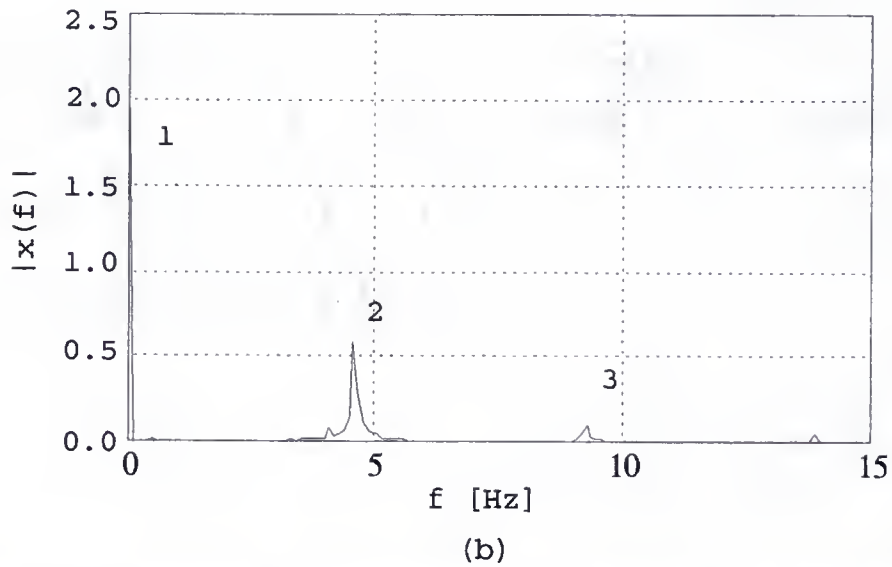
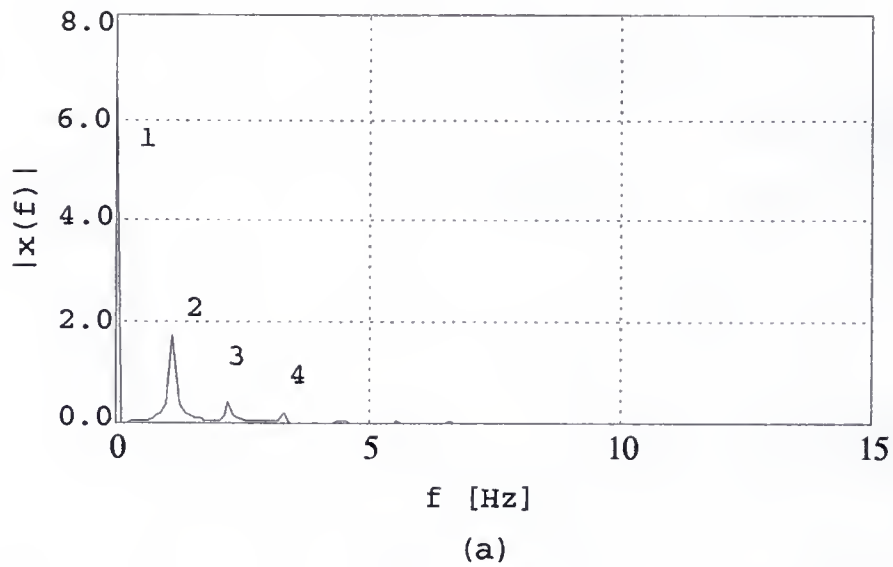


Figure 3.39 Spectrum Analysis of One Degree of Freedom System
(a) Flight Mode, (b) Ground Mode

Table 3.27 Frequencies From Period Diagram

Mode	Frequency (Hz)
Ground	4.6296
Flight	1.1074

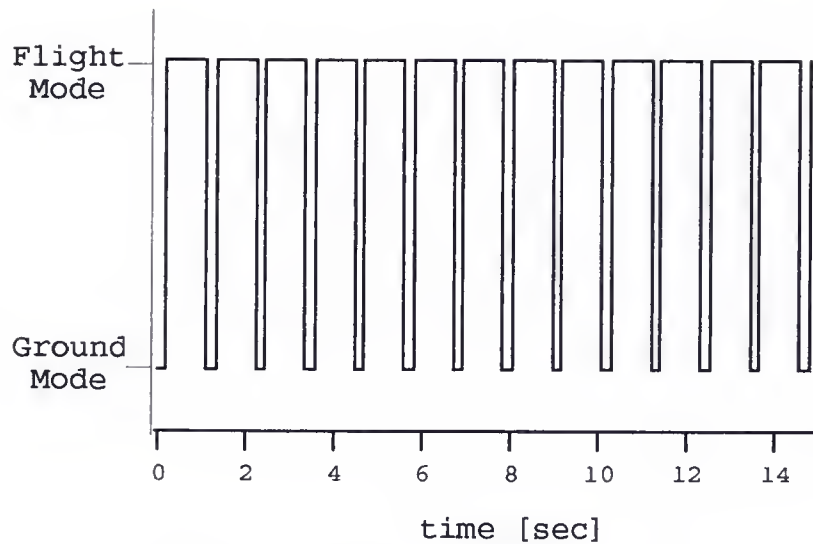


Figure 3.40 Period Diagram

3.6 Summary

Before proceeding with the summary, it is necessary to note that the ground mode of a one degree of freedom system begins as soon as the spring touches the surface. However, the natural frequency of system c is obtained from the equation of motion based on the equilibrium position of the system. There exists a difference between the ground contact reference and the equilibrium position of the system. This is a static displacement (δ_{st}). Thus, the system has to move the static displacement to reach the equilibrium position after the beginning of the ground mode. At the end of the ground mode, the system moves up the static displacement beyond the equilibrium position before reaching the ground contact reference. It is assumed that the velocity of the system during that period is $\sqrt{2gh}$. The static displacement

can be expressed as

$$\delta_{st} = \frac{Mg}{K} \quad (3.4)$$

Therefore, the time required for the system to travel the static displacement at the beginning and the end of the ground mode is

$$t_{\delta_{st}} = \frac{2\delta_{st}}{\sqrt{2gh}} \quad (3.5)$$

The ground mode frequency (ω_g) obtained from the spectrum analysis must be corrected to include the time required for the system to travel the static displacement. The corrected frequency of the ground mode with consideration of the equilibrium point can be obtained as

$$\Omega_g = \frac{1}{\frac{1}{\omega_g} - t_{\delta_{st}}} \quad (3.6)$$

This corrected frequency (Ω_g) can be used to get relationships with dominating frequencies of each mode and the natural frequency of system *c*.

The results of spectrum analyses for two degree of freedom and one degree of freedom bouncing systems with different system parameters and initial conditions were obtained. Those results are now summarized to establish relationships between the dominant frequencies and the natural mode frequencies.

Case of 570N/m as Foot Stiffness ($h = 0.2\text{m}$)

At first, the ground mode frequency of one degree of freedom system ($\Omega_{sg2} = 1.6602 \text{ Hz}$ from Table 3.4) is considered. The static displacement is 0.1893m by Equation (3.4), and the traveling time is 0.1911 second by Equation (3.5). Thus, the ground mode frequency (Ω_g) with consideration of the equilibrium position is 2.4317 Hz obtained from Equation (3.6), which is approximately twice the natural frequency of system c ($\omega_c = 1.1457 \text{ Hz}$ from Table 3.1), i.e., $\Omega_g \cong 2\omega_c$. The natural frequency of system c is very close to the most dominant ground mode frequency of the two degree of freedom system ($\Omega_{dg2} = 1.1719 \text{ Hz}$ from Table 3.2).

The dominant frequencies for the one degree of freedom system without separation are 0.9766 Hz (Ω_{st2}) and 1.9351 Hz (Ω_{st3}) (Table 3.3). If the most dominant ground mode frequency of the two degree of freedom system is subtracted from $2\Omega_{st3}$, then the result is 2.6983 Hz and it is close to the most dominant flight mode frequency of the two degree of freedom system ($\Omega_{df4} = 2.8320 \text{ Hz}$ from Table 3.2). In short, for the flight mode, the frequency relationship can be expressed as $\Omega_{df4} \cong 2\Omega_{st3} - \Omega_{dg2}$.

Case of 3670N/m as Foot Stiffness ($h = 0.2\text{m}$)

For the two degree of freedom system, the most dominant ground mode frequency (Ω_{dg2}) and the most dominant flight mode frequency (Ω_{df3}) are identically 2.2461 Hz (Table 3.6).

For the one degree of freedom system, the most dominant flight mode frequency is 2.4414 Hz (Ω_{sf2}) and the most dominant ground mode frequency is 4.9805 Hz (Ω_{sg2}) from Table 3.8. The most dominant ground mode frequency (Ω_{sg2}) is approximately twice the most dominant flight mode frequency (Ω_{sf2}), i.e., $\Omega_{sg2} \approx 2\Omega_{sf2}$.

The dominant frequencies for the one degree of freedom system without separation are 1.6602 Hz (Ω_{st2}), 3.3203 Hz (Ω_{st3}), and 4.9805 Hz (Ω_{st4}) (Table 3.7). If the most dominant ground mode frequency of the two degree of freedom system is subtracted from $2\Omega_{st3}$, then the result is 4.3945 Hz and is close to two times of the most dominant flight mode frequency of the two degree of freedom system ($\Omega_{df3} = 2.2461$ Hz from Table 3.6). In short, for the flight mode, the frequency relationship can be expressed as $2\Omega_{df3} \approx 2\Omega_{st3} - \Omega_{dg2}$.

Case of 445N/m as Foot Stiffness ($h = 0.5m$)

For the two degree of freedom system, the most dominant flight mode frequency (Ω_{df3}) is 1.8555 Hz and the most dominant ground mode frequency (Ω_{dg2}) is 1.2695 Hz (Table 3.11). For the one degree of freedom system, the most dominant flight mode frequency (Ω_{sf2}) and the most dominant ground mode frequency (Ω_{sg2}) are identically 1.5625 Hz from Table 3.13.

The dominant frequencies for the one degree of freedom system without separation are 0.7813 Hz (Ω_{st2}) and 1.5625 Hz

(Ω_{st3}) (Table 3.12). If the most dominant ground mode frequency of the two degree of freedom system is subtracted from $2\Omega_{st3}$, then the result is 1.8555 Hz and is the same as the most dominant flight mode frequency of the two degree of freedom system (Ω_{df3}). In short, for the flight mode, the frequency relationship can be expressed as $\Omega_{df3} = 2\Omega_{st3} - \Omega_{dg2}$. Also the average of the most dominant flight mode frequency and the most dominant ground mode frequency is 1.5625 Hz.

Case of 1860N/m as Foot Stiffness ($h = 0.5m$)

The most dominant ground mode frequency of the one degree of freedom system (Ω_{sg2}) is 3.6133 Hz from Table 3.17. By using Equation (3.4), Equation (3.5), and Equation (3.6), the corresponding ground mode frequency (Ω_g) with consideration of the equilibrium position is obtained as 4.1709 Hz which is approximately twice the natural frequency of system c ($\omega_c = 2.0696$ Hz from Table 3.10), i.e., $\Omega_g \approx 2\omega_c$. The natural frequency of system c is very close to the most dominant ground mode frequency of the two degree of freedom system ($\Omega_{dg2} = 2.2461$ Hz from Table 3.15). The dominant frequencies for the one degree of freedom system without separation are 1.0742 Hz (Ω_{st2}), 2.1484 Hz (Ω_{st3}), and 3.3203 Hz (Ω_{st4}) (Table 3.16). If Ω_{st1} is subtracted from Ω_{st4} , then the result is the most dominant ground mode frequency of the two degree of freedom system, i.e., $\Omega_{dg2} = \Omega_{st4} - \Omega_{st2} = 2.2461$ Hz.

The most dominant flight frequency is 1.5625 Hz for both one and two degree of freedom systems.

Case of 2010N/m as Foot Stiffness ($h = 1.0\text{m}$)

For the ground mode of the one degree of freedom system, the most dominant frequency (Ω_{sg2}) is 3.9063 Hz from Table 3.22. If the equilibrium position and the ground contact reference at the beginning and the end of the ground mode is considered, the most dominant frequency (Ω_{sg2}) becomes 4.3141 Hz by using Equation (3.4), Equation (3.5), and Equation (3.6). This corrected ground mode frequency ($\Omega_g = 4.3141$ Hz) based on the equilibrium position is twice the natural frequency of system c ($\omega_c = 2.1514$ Hz from Table 3.10), i.e., $\Omega_g \approx 2\omega_c$. For the two degree of freedom system, the most dominant ground mode frequency (Ω_{dg3}) is 2.5391 Hz and the second most dominant ground mode frequency (Ω_{dg2}) is 2.2461 Hz from Table 3.20. In this case, the natural frequency of system c (ω_c) is very close to the second most dominant ground mode frequency (Ω_{dg2}) of the two degree of freedom system, i.e., $\Omega_{dg2} \approx \omega_c$. The dominant frequencies for the one degree of freedom system without separation are 0.8789 Hz (Ω_{st2}), 1.7578 Hz (Ω_{st3}), 2.5391 Hz (Ω_{st4}), and 3.4180 Hz (Ω_{st5}) from Table 3.21. The most dominant ground mode frequency (Ω_{dg3}) for the two degree of freedom system is equal to Ω_{st4} .

For the flight mode of the two degree of freedom system, the most dominant frequency is 1.1719 Hz (Ω_{df2}) from Table

3.20. The most dominant flight mode frequency is 1.0742 Hz (Ω_{sf2}) and the second most dominant flight mode frequency is 2.2461 Hz (Ω_{sf3}) for the one degree of freedom system. Ω_{df2} is the same as the difference between Ω_{sf2} and Ω_{sf3} , i.e., $\Omega_{df2} = \Omega_{sf3} - \Omega_{sf2}$.

Case of 2745N/m as Foot Stiffness ($h = 1.0m$)

For the ground mode of the one degree of freedom system, the most dominant frequency (Ω_{sg2}) is 4.5898 Hz from Table 3.26. With consideration of the equilibrium position and the ground contact reference at the beginning and the end of the ground mode, the most dominant frequency (Ω_{sg2}) becomes 4.9981 Hz by using Equation (3.4), Equation (3.5), and Equation (3.6). This ground mode frequency (Ω_g) based on the equilibrium position is twice the natural frequency of *system c* ($\omega_c = 2.5142$ Hz from Table 3.10), i.e., $\Omega_g \approx 2\omega_c$. For the two degree of freedom system, the most dominant ground mode frequency (Ω_{dg3}) is 3.6133 Hz and the second most dominant ground mode frequency (Ω_{dg2}) is 2.6397 Hz from Table 3.24. In this case, the natural frequency of *system c* (ω_c) is very close to the second most dominant ground mode frequency (Ω_{dg2}) of the two degree of freedom system, i.e., $\Omega_{dg2} \approx \omega_c$. The dominant frequencies for the one degree of freedom system without separation are 0.8789 Hz (Ω_{st2}), 1.7578 Hz (Ω_{st3}), 2.7344 Hz (Ω_{st4}), and 3.6133 Hz (Ω_{st5}) from Table 3.25. The most dominant ground mode frequency (Ω_{dg3}) for the two degree of freedom system is equal to Ω_{st5} .

For the flight mode of the two degree of freedom system, the most dominant frequency is 1.1719 Hz (Ω_{df2}) from Table 3.24. The most dominant flight mode frequency is 1.0742 Hz (Ω_{sf2}) and the second most dominant flight mode frequency is 2.2461 Hz (Ω_{sf3}) for the one degree of freedom system. Ω_{df2} is same as the difference between Ω_{sf2} and Ω_{sf3} , i.e., $\Omega_{df2} = \Omega_{sf3} - \Omega_{sf2}$.

Two most common frequency relationships can be found among chaotic systems. The first one is that the most (or the second most) dominant ground mode frequency of a two degree of freedom system is equal to the natural frequency of system c, which is equal to half of the most dominant ground mode frequency of a corresponding one degree of freedom system. The second one is that the most dominant frequencies are identical for both the ground mode and the flight mode either for corresponding one degree of freedom systems or two degree of freedom systems. Even though there might exist different criteria of chaotic bouncing motions other than these two common frequency relationships, one can use these two frequency relationships to check whether or not a system with selected parameters has chaotic bouncing motions.

In order to verify the frequency relationships, 2415N/m is used as the foot stiffness with a 0.5m dropping height. The most dominant ground mode frequency of one degree of freedom system (Ω_{sg}) is 4.1991 Hz and the corresponding frequency (Ω_g) with consideration of the equilibrium position becomes 4.7708 Hz. The most dominant ground mode frequency

of the two degree of freedom system (Ω_{dg}) is 2.4414 Hz while the natural frequency of system c (ω_c) is 2.3582 Hz. Since Ω_{dg} is approximately equal to ω_c or $\frac{\Omega_g}{2}$, the first frequency relationship holds in this case. According to Figure 3.3, in fact, the system has chaotic bouncing motion since the sum of standard deviations and the area are high.

The value of 1815N/m as foot stiffness is selected with a 0.2m dropping height to check the established frequency relationships. The most dominant flight mode frequencies of the corresponding one degree of freedom system and the two degree of freedom system are identically 2.4414 Hz. The most dominant ground mode frequency of the one degree of freedom system (Ω_{sg}) is 3.3203 Hz and the corresponding frequency (Ω_g) with consideration of the equilibrium position becomes 4.1468 Hz. The most dominant ground mode frequency of the two degree of freedom system (Ω_{dg}) is 1.8555 Hz while the natural frequency of system c (ω_c) is 2.0444 Hz. The system does not fall in the first frequency relationship since Ω_{dg} is not equal and/or close to ω_c . Neither can the second frequency relationship be found for this system. In fact, the system with chosen system parameters and initial conditions does not have chaotic bouncing behavior since the sum of standard deviations and area are small in Figure 3.2.

With these two more examples, it can be concluded that the most significant system mode for chaos is not the flight mode but the ground mode as in rigid body motion.

CHAPTER 4 ELIMINATION OF CHAOS IN TWO DEGREE OF FREEDOM SYSTEMS

4.1 Background

Previous chapters have presented the conditions for chaotic bouncing motions depending on the initial conditions and system parameters. It was assumed that there is no energy dissipation throughout the investigation of the system behavior.

Since the chaotic behavior is not desirable, elimination methods for this objectionable characteristics of the system should be provided. This can be done by varying system parameters or by introducing damping elements to the system. However, with damping elements, the loss of energy must be compensated by external energy sources in order to make the system keep bouncing.

4.2 Spring Selections

The sum of standard deviations and area in a phase plane plot have been discussed in Chapter 3 (Figure 3.2, Figure 3.3, and Figure 3.4). The range is set so that the data for the ground and the flight modes can be separated easily. These two results are almost identical. If the system has chaotic motions, the sum of standard deviations and the area

are increased. On the other hand, the sum of the standard deviations or the area becomes small for the repeated bouncing motions.

The range for the foot stiffness (K_2) is a system parameter which can easily be selected for the repeated bouncing motion from Figure 3.2 when the dropping height (h) is 0.2m. The foot stiffness, which makes the sum of standard deviations or area small in Figure 3.2, can also be chosen for obtaining repeated bouncing motion. For this case, the range of foot stiffness is approximately between 1000N/m and 2000N/m. For example, if 1500N/m is selected as the foot stiffness, there are repeated bouncing motions. Figure 4.1 illustrates that the chaotic behavior exhibited in Figure 2.6 can thus be avoided by an appropriate choice of system parameters.

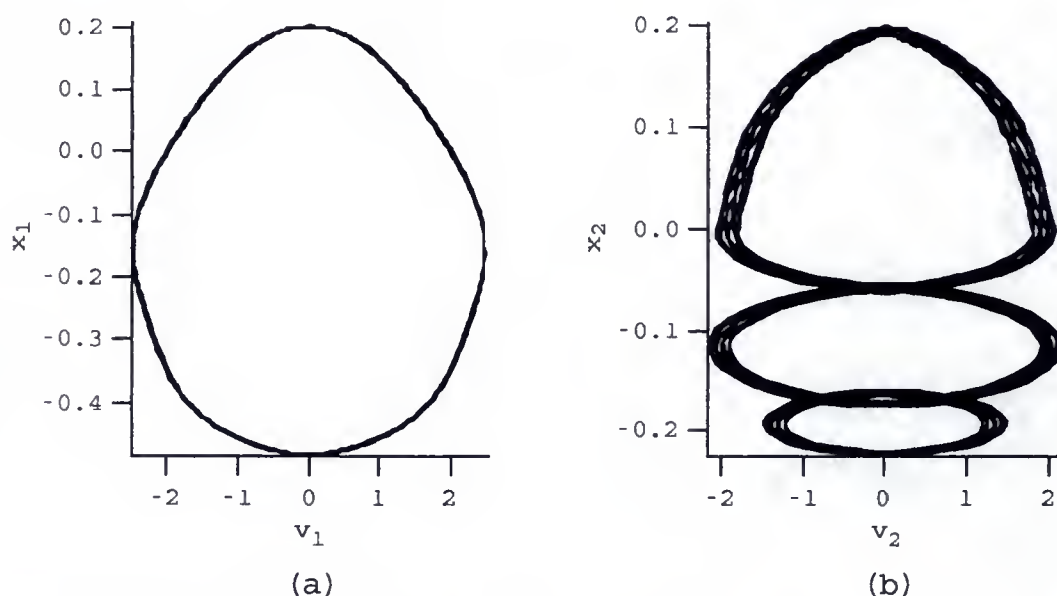


Figure 4.1 Phase Planes with $h = 0.2\text{m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_1 = 1000\text{N/m}$, and $K_2 = 1500\text{N/m}$
 (a) Body Phase Plane, (b) Foot Phase Plane

The foot stiffness (K_2), which makes the sum of standard deviations or area small, can be obtained from Figure 3.3 for the case of 0.5m dropping height and from Figure 3.4 for the case of 1.0m dropping height. However, there are no selective ranges for the system parameter in Figure 3.3 and Figure 3.4. For example, if the foot stiffness is chosen among the values of 195N/m, 295N/m, 550N/m, and 3080N/m, the system would have repeated bouncing motions when dropping height is 0.5m. For the case of 1.0m dropping height, 125N/m, 165N/m, 245N/m, 430N/m, and 3775N/m are the values of the foot stiffness to be selected for repeated bouncing motions. Figure 4.2 and Figure 4.3 display the repeated bouncing motions for the cases of 0.5m and 1.0m dropping heights with 550N/m and 3775N/m as selected foot stiffness, respectively.

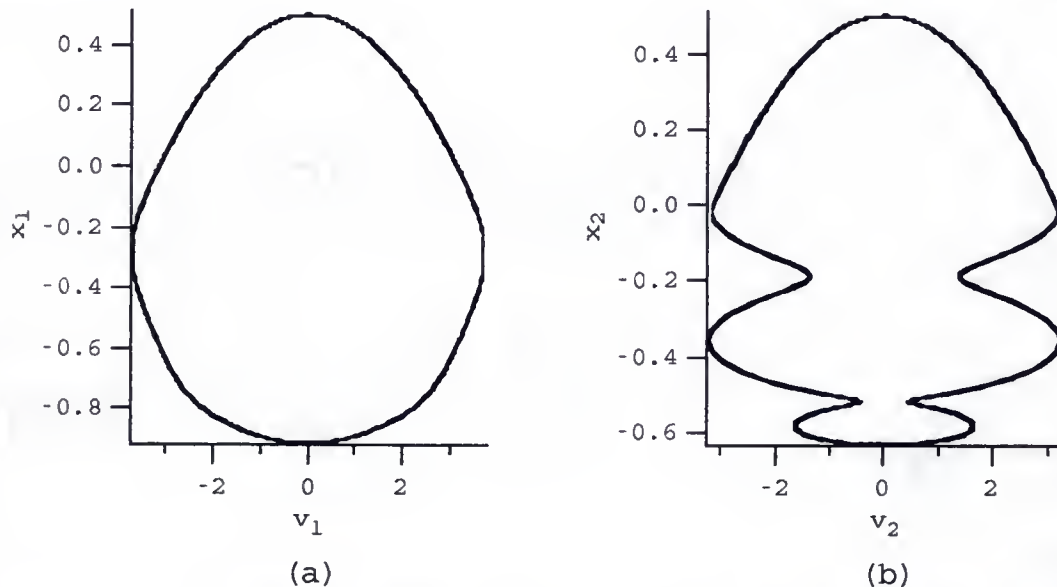


Figure 4.2 Phase Planes with $h = 0.5\text{m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_1 = 1000\text{N/m}$, and $K_2 = 550\text{N/m}$
 (a) Body Phase Plane, (b) Foot Phase Plane

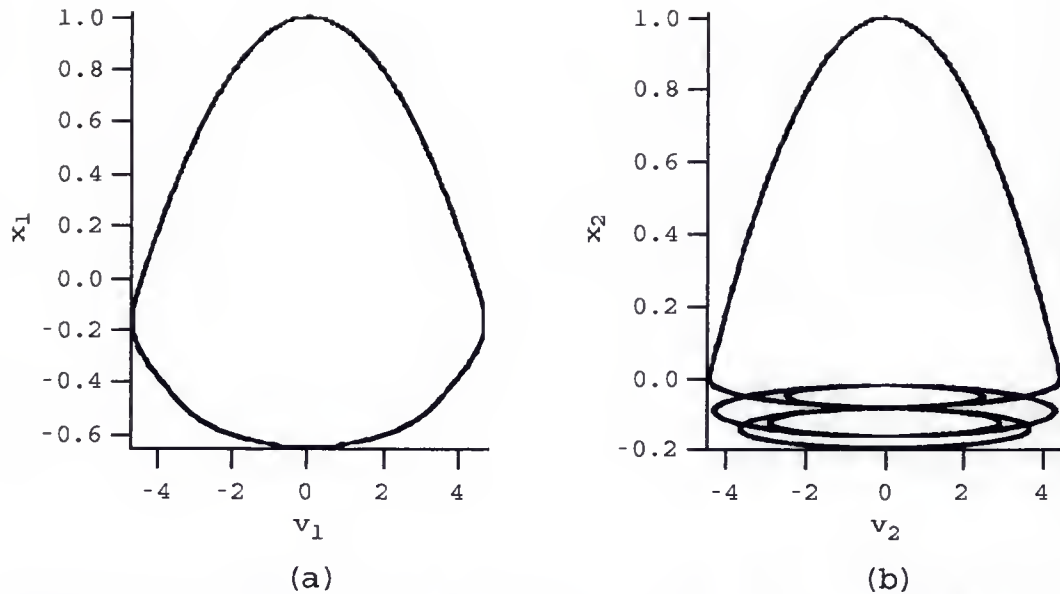


Figure 4.3 Phase Planes with $h = 1.0\text{m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$,
 $K_1 = 1000\text{N/m}$, and $K_2 = 3775\text{N/m}$
 (a) Body Phase Plane, (b) Foot Phase Plane

4.3 Use of Damping Elements

It has been illustrated that chaotic system behavior can be eliminated by properly choosing system parameters under the assumption of no energy dissipation. However, it may be impossible to build a physical system without any energy dissipation. One of the major energy dissipation sources is friction which can be modelled as a damping element. Although the actual description of the damping force is difficult, ideal damping models often result in satisfactory prediction of the response. Of these models, a viscous damping force, proportional to velocity, leads to the simplest mathematical treatment. Therefore, viscous damping would be used as the energy dissipation source and the application of it may provide a potential elimination method

for chaotic bouncing motions.

There are three different ways to add damping elements to two degree of freedom systems: damping element to the foot, damping element to the body, and damping elements to both the body and the foot. However, only the first case will be discussed, since an addition of a damping element to the foot could contribute for the elimination of chaotic behavior. If the viscous damping constant (C_2) is too small, addition of a damping element may not help to eliminate chaotic bouncing motions. The larger damping constant the system has, the more likely chaotic bouncing motions can be removed. However, if the damping coefficient is too high, the system may not bounce at all. If the damping constant is bigger than $20\text{N}\cdot\text{sec}/\text{m}$ with $570\text{N}/\text{m}$ of foot stiffness and 0.2m of dropping height, the system never leaves the ground after releasing from the given height without any external energy source. When the system starts to have this behavior with a damping element, the damping constant is called *critical system damping for bouncing*. Critical system damping coefficients for bouncing (C_{cr}) are shown in Table 4.1 for the previously discussed cases. If the damping constant is smaller than critical system damping coefficient for bouncing, the system has both ground mode and flight mode initially. However, the system eventually would have only ground mode due to gradual energy loss, and finally the system would stop without any motion. Therefore, in any cases, the dissipated energy due to damping must be

compensated by external energy sources for continued bouncing motions.

Table 4.1 Critical System Damping Coefficients for the Previously Discussed Cases

Foot Stiffness (K_2) [N/m]	Dropping Height (h) [m]	Damping Constant (C_{cr}) [N•sec/m]
570	0.2	20
3670	0.2	420
445	0.5	28
1860	0.5	260
2010	1.0	470
2745	1.0	745

In order to see the effects of changes of damping constants, the bouncing system with 1860N/m of foot stiffness and 0.5m of dropping height, which is the most chaotic system for 0.5m of dropping height, will be used as an example. Phase planes without damping are shown in Figure 2.8. At first, 5N•sec/m is used as a damping coefficient (Figure 4.1). For the body phase plane, most of high frequency motions are removed. Although, with this damping, the motion of the foot still displays a little bit of chaos. If the damping coefficient is increased to 10N•sec/m (Figure 4.5) and 25N•sec/m (Figure 4.6), chaotic motions of the foot can be eliminated more. However, bouncing motions would also be decreased and motions of the system will be stopped due to energy dissipation. In fact, with 10N•sec/m of damping constant, the system has 14 bouncing cycles while the system has only 6 bouncing cycles with 25N•sec/m of damping

constant. If the damping constant is between $100\text{N}\cdot\text{sec}/\text{m}$ (Figure 4.7) and $260\text{N}\cdot\text{sec}/\text{m}$ (Figure 4.8), then the system would bounce only once after releasing from 0.5m of dropping height. Thus, there is a tradeoff. With a high damping constant, it is easy to eliminate chaotic bouncing motions. However, it is necessary to have a powerful external energy source, which can produce required power at once. On the other hand, chaotic bouncing motions may still exist with a very low damping constant.

Therefore, further research for an external energy source should be combined with damping elements to build a practical legged system.

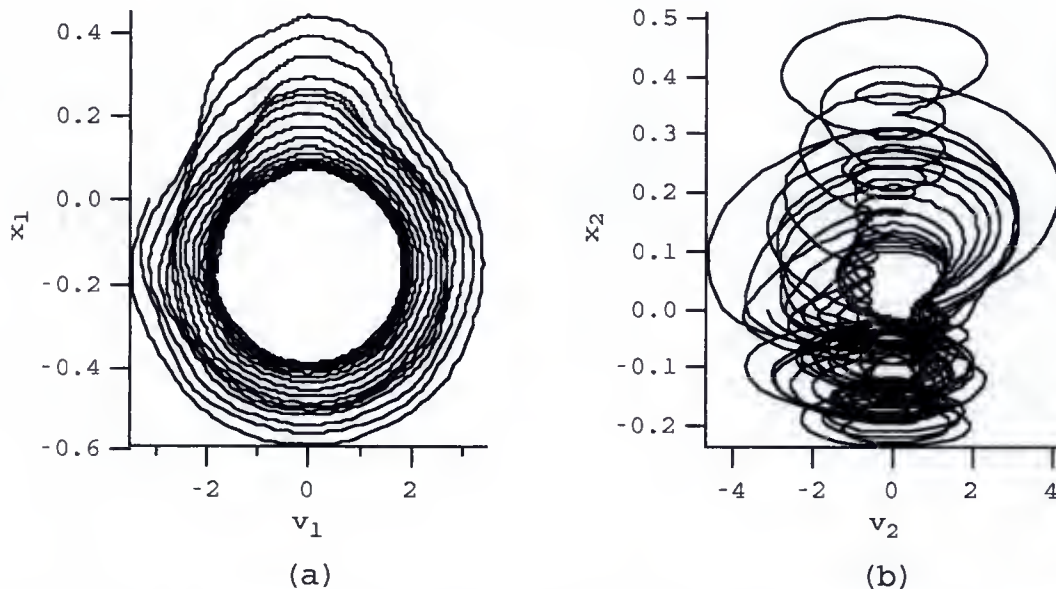


Figure 4.4 Phase Planes of Two Degree of Freedom System with $h = 0.5\text{m}$, $K_1 = 1000\text{N}/\text{m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_2 = 1860\text{N}/\text{m}$, and $C_2 = 5\text{ N}\cdot\text{sec}/\text{m}$

(a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

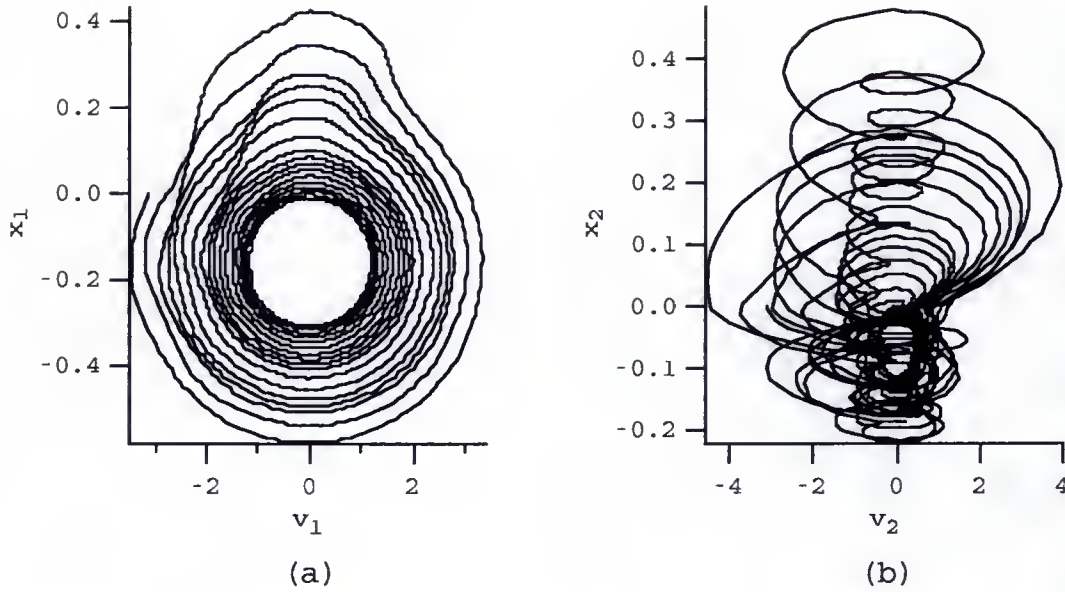


Figure 4.5 Phase Planes of Two Degree of Freedom System with $h = 0.5\text{m}$, $K_1 = 1000\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_2 = 1860\text{N/m}$, and $C_2 = 10 \text{ N}\cdot\text{sec/m}$

(a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

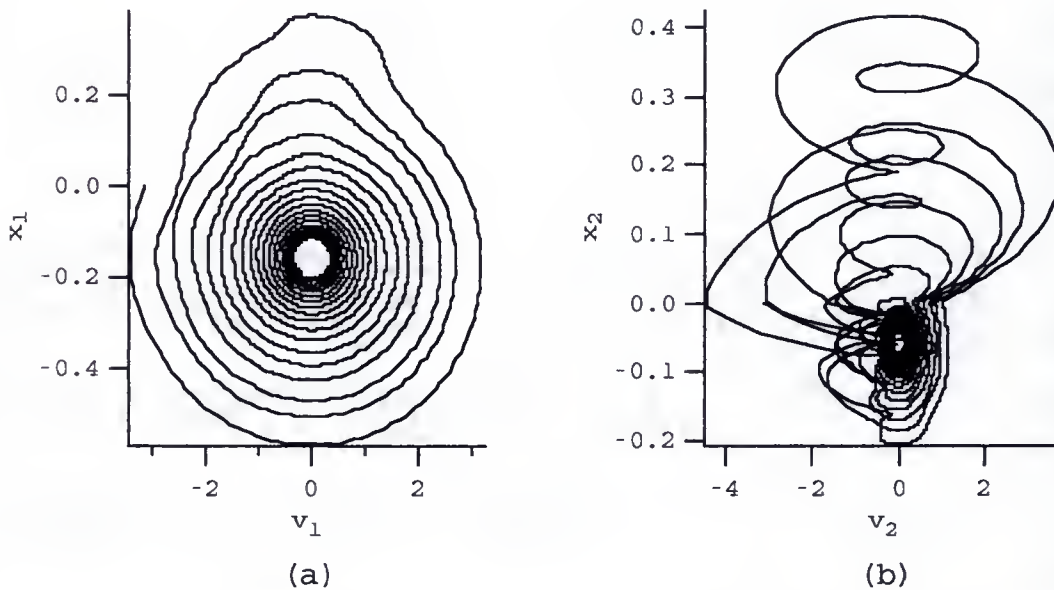


Figure 4.6 Phase Planes of Two Degree of Freedom System with $h = 0.5\text{m}$, $K_1 = 1000\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_2 = 1860\text{N/m}$, and $C_2 = 25 \text{ N}\cdot\text{sec/m}$

(a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

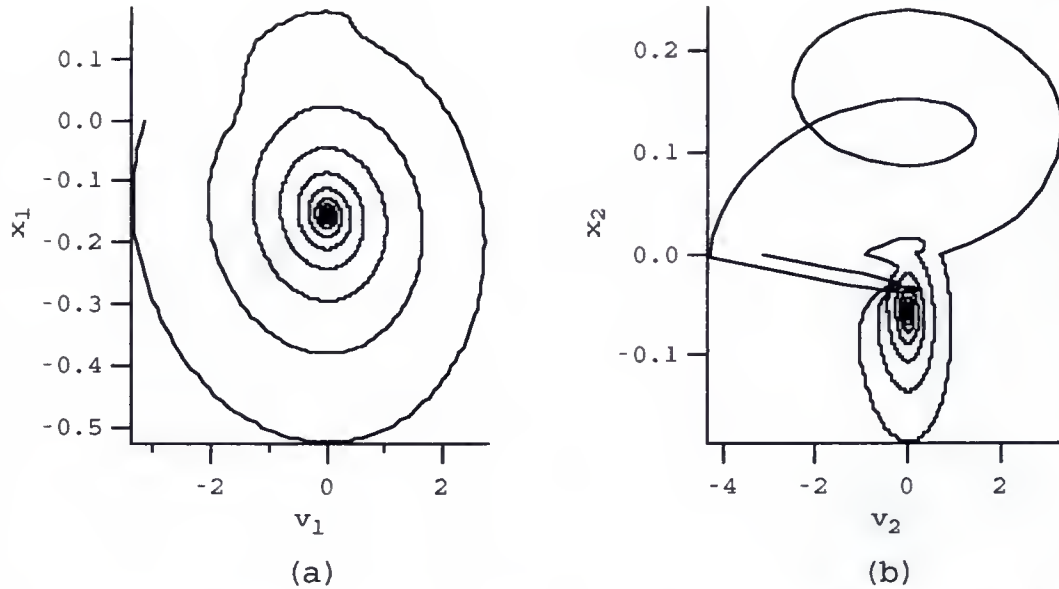


Figure 4.7 Phase Planes of Two Degree of Freedom System with $h = 0.5\text{m}$, $K_1 = 1000\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_2 = 1860\text{N/m}$, and $C_2 = 100 \text{ N}\cdot\text{sec/m}$

(a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

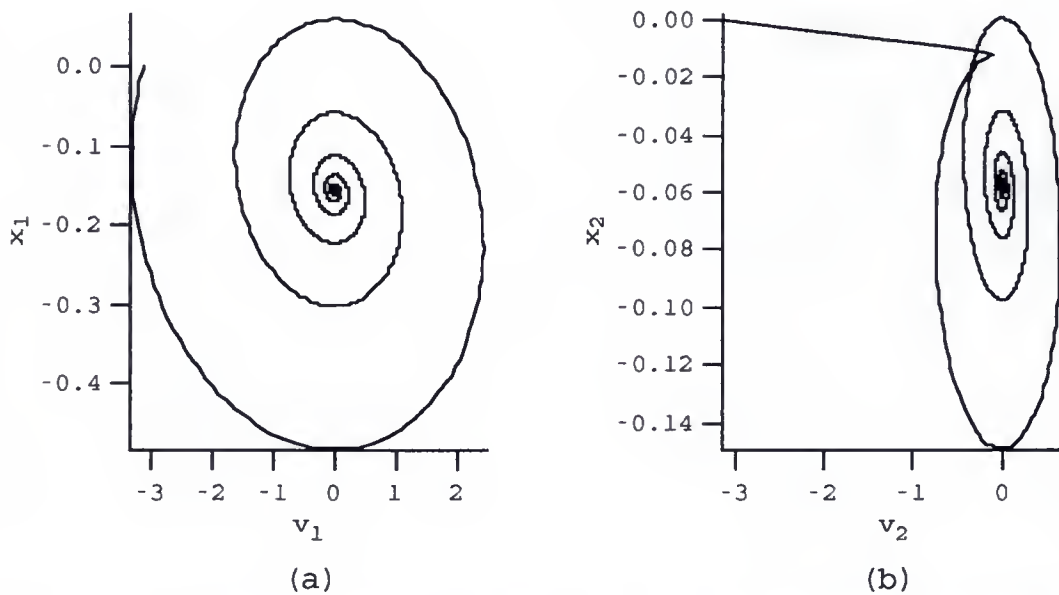


Figure 4.8 Phase Planes of Two Degree of Freedom System with $h = 0.5\text{m}$, $K_1 = 1000\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, $K_2 = 1860\text{N/m}$, and $C_2 = 260 \text{ N}\cdot\text{sec/m}$

(a) Body Phase Plane (M_1), (b) Foot Phase Plane (M_2)

CHAPTER 5 CONCLUSIONS AND RECOMMENDATIONS

Chaotic behavior of simplified bouncing systems has been observed depending on not only system parameters but also initial conditions. Chaotic bouncing systems with different parameters and initial conditions have been analyzed to find some prediction schemes for chaos. A Fast Fourier Transform has been used for the spectrum analysis. Data sets have been separated in order to obtain dominating frequencies for the ground and flight modes of the system. Frequency relationships between dominating frequencies of each system mode and natural frequencies of some linear oscillatory systems have shown that the rigid body ground mode is the most important for chaotic behavior of two degree of bouncing systems. Methods for elimination of chaos have also been discussed.

Although the research results demonstrate that frequency relationships can be used to predict chaotic bouncing motions, it does not provide, by any means, a general design technique for nonlinear intermittent contact problems. However, it is hoped that this work will be considered as a viable method for the design and analysis for various intermittent contact problems such as bilinear oscillatory systems.

One immediate extension of this work is to use a rotating unbalance as external power source. A counterrotating eccentric weight exciter, along with damping elements to remove chaos, may be used for maintaining steady state bouncing motions by adjusting an eccentric mass with eccentricity which is rotating with angular velocity, since it provides mainly vertical excitation while horizontal excitation is minimized.

APPENDIX A EFFECTS OF TIME STEP SIZE

Numerical integrations have been carried out to study the bouncing systems. Small time steps must be used to get the correct responses of two degree of freedom bouncing systems, since there might be chaos which is very sensitive. If the time step is not small enough, the numerical solutions can lead to spurious existence of chaos. For example, 0.02 second is used in Figure A.1 as time step for the body phase plane plot with $K_1 = 1000\text{N/m}$, $K_2 = 1860\text{N/m}$, $M_1 = 10\text{Kg}$, $M_2 = 1\text{Kg}$, and $h = 0.5\text{m}$. With the same system parameters, Figure A.2 displays the body phase plane when the time step size is 0.001 second. Even though both phase planes do not have repeated bouncing motions, it proves that if the time step is not small enough, the system response by the numerical integrations may lead to false conclusion of chaos or subharmonics.

It is confirmed that one millisecond (0.001 second) is small enough to be used as time step throughout the investigation of chaotic behavior of bouncing systems by reducing it to ten micro-second (0.00001 second). In fact, the time responses by numerical integrations with one millisecond and ten micro-second for chaotic bouncing systems are identical.

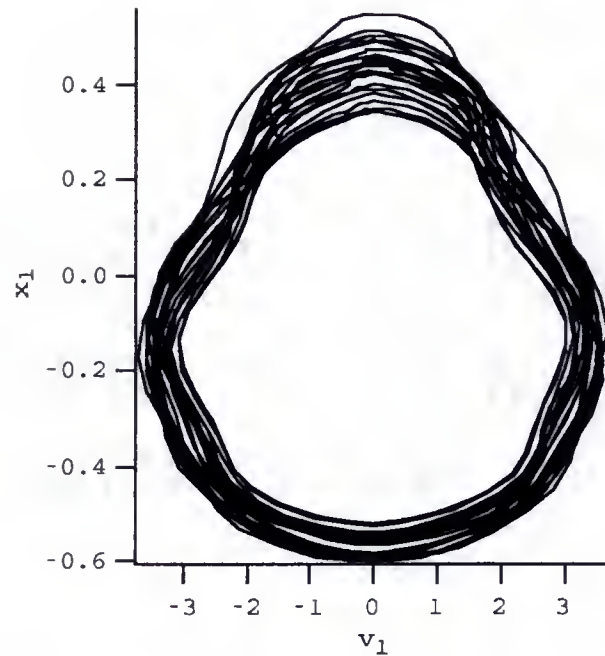


Figure A.1 Body Phase Plane of Two Degree of Freedom System
with 0.05 second as time step
($K_1=1000\text{N/m}$, $K_2=1860\text{N/m}$, $M_1=10\text{Kg}$, $M_2=1\text{Kg}$, and $h=0.5\text{m}$)

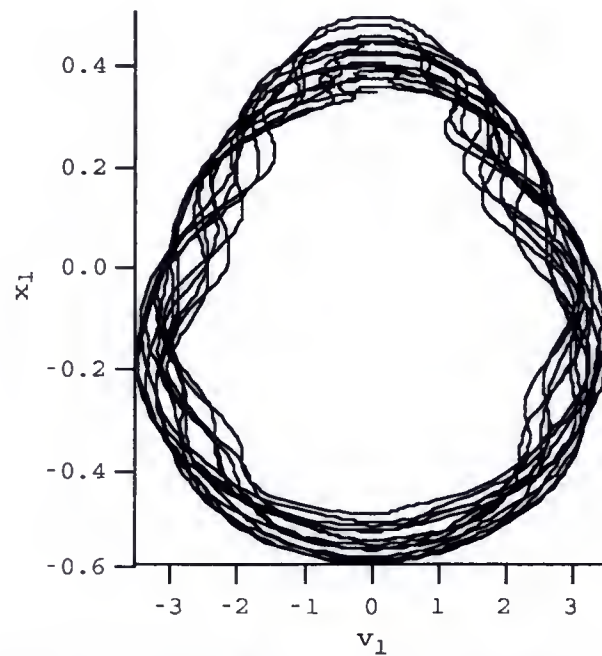


Figure A.2 Body Phase Plane of Two Degree of Freedom System
with 0.001 second as time step
($K_1=1000\text{N/m}$, $K_2=1860\text{N/m}$, $M_1=10\text{Kg}$, $M_2=1\text{Kg}$, and $h=0.5\text{m}$)

APPENDIX B PERIODS FROM PHASE PLANE

The periods of the flight and the ground modes for one degree of freedom system can be obtained from the phase plane.

The displacement (x) is measured from the ground contact reference which is the position of the body at the instant when the spring first reaches the ground. However, the equilibrium must be entered for the phase plane trajectories. Thus, the system has to travel a little further, as much as the static displacement¹, for the body to reach the equilibrium position after the spring end touches the surface (Figure B.1).

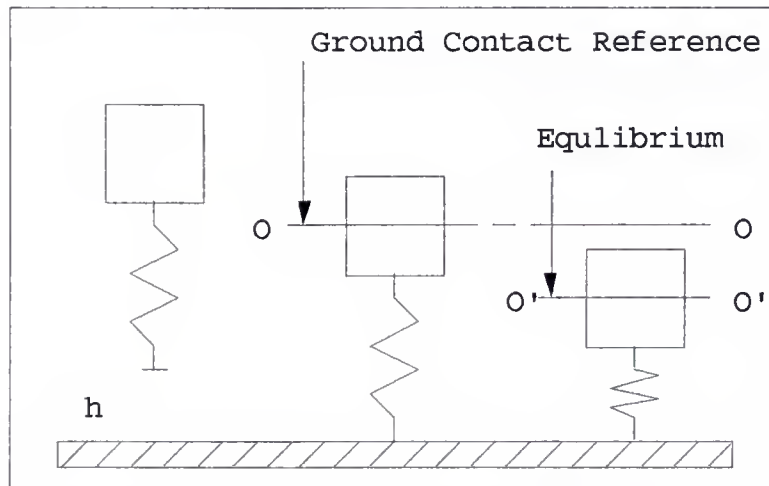


Figure B.1 The Ground Contact Reference and Equilibrium

¹ static displacement (x_g) = $\frac{Mg}{K}$, where M is mass, g is the gravity, and K is spring coefficient.

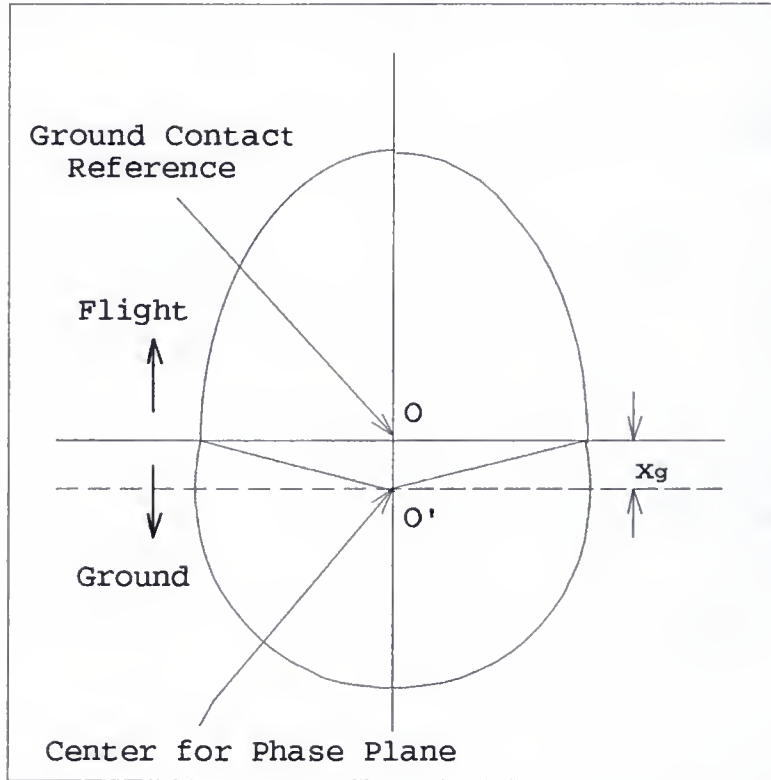


Figure B.2 A Phase Plane for One Degree of Freedom System
(x: vertical axis, v/ω_n : horizontal axis)

From Figure B.2, the equation for the angle from the beginning and the end of ground mode can be expressed as

$$\begin{aligned}\omega_n t_g &= \pi + 2\arctan\left(\frac{x_g}{\frac{\sqrt{2gh}}{\omega_n}}\right) \\ &= \pi + 2\arctan\left(\sqrt{\frac{Mg}{2Kh}}\right)\end{aligned}\quad (B.1)$$

where, t_g is the period for the ground mode, K is the stiffness, h is the dropping height, ω_n is the natural frequency. Therefore, the ground mode period can be expressed as

$$t_g = \sqrt{\frac{M}{K}} \left[\pi + 2\arctan\left(\sqrt{\frac{Mg}{2Kh}}\right) \right] \quad (B.2)$$

For the flight mode, it is pure free fall motion in the gravitational field with $\sqrt{2gh}$ as the initial velocity and zero as initial displacement. Thus, the flight mode period is

$$t_f = 2\sqrt{\frac{2h}{g}} \quad (\text{B.3})$$

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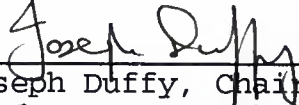
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BIOGRAPHICAL SKETCH

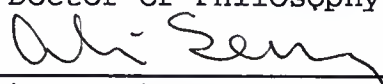
Chi-Wook Lee was born in Suwon, Korea, on November 18th, 1957. After graduating from Seoul High School he began college studies at Han-Yang University. He received his Bachelor of Engineering degree in 1981. He began graduate studies at University of Wisconsin-Madison after his military service in the Air Force. After his Master of Science degree, he continued his graduate studies at Carnegie-Mellon University between 1985 and 1987. He continued his graduate studies at University of Florida in 1987.

He will begin his professional career at University of Michigan-Flint in 1991.

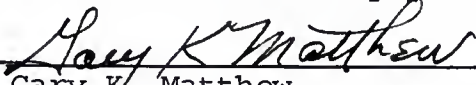
I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.


Joseph Duffy, Chairman
Graduate Research Professor of
Mechanical Engineering

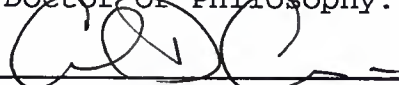
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Ali A. Seireg, Cochairman
Ebaugh Professor of
Mechanical Engineering

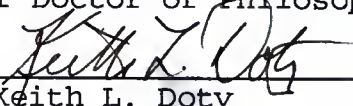
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Gary K. Matthew
Associate Professor of
Mechanical Engineering

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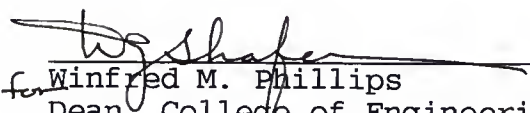

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.


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This dissertation was submitted to the Graduate Faculty of the College of Engineering and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1991


for Winfred M. Phillips
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